This is a digital copy of a book that was preserved for generations on library shelves before it was carefully scanned by Google as part of a project to make the world's books discoverable online.

It has survived long enough for the copyright to expire and the book to enter the public domain. A public domain book is one that was never subject to copyright or whose legal copyright term has expired. Whether a book is in the public domain may vary country to country. Public domain books are our gateways to the past, representing a wealth of history, culture and knowledge that's often difficult to discover.

Marks, notations and other marginalia present in the original volume will appear in this file - a reminder of this book's long journey from the publisher to a library and finally to you.

Usage guidelines

Google is proud to partner with libraries to digitize public domain materials and make them widely accessible. Public domain books belong to the public and we are merely their custodians. Nevertheless, this work is expensive, so in order to keep providing this resource, we have taken steps to prevent abuse by commercial parties, including placing technical restrictions on automated querying.

We also ask that you:

+ *Make non-commercial use of the files* We designed Google Book Search for use by individuals, and we request that you use these files for personal, non-commercial purposes.

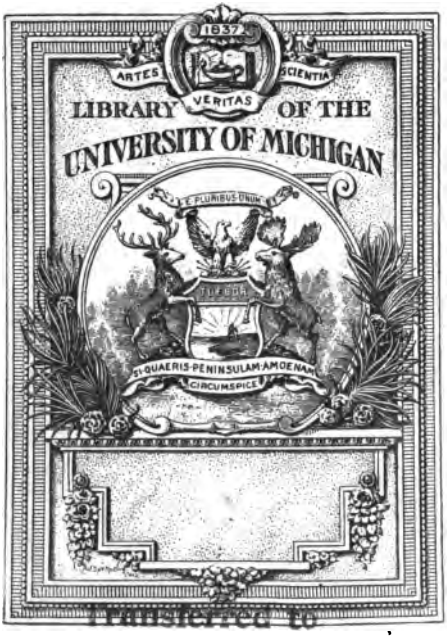
+ *Refrain from automated querying* Do not send automated queries of any sort to Google's system: If you are conducting research on machine translation, optical character recognition or other areas where access to a large amount of text is helpful, please contact us. We encourage the use of public domain materials for these purposes and may be able to help.

+ *Maintain attribution* The Google "watermark" you see on each file is essential for informing people about this project and helping them find additional materials through Google Book Search. Please do not remove it.

+ *Keep it legal* Whatever your use, remember that you are responsible for ensuring that what you are doing is legal. Do not assume that just because we believe a book is in the public domain for users in the United States, that the work is also in the public domain for users in other countries. Whether a book is still in copyright varies from country to country, and we can't offer guidance on whether any specific use of any specific book is allowed. Please do not assume that a book's appearance in Google Book Search means it can be used in any manner anywhere in the world. Copyright infringement liability can be quite severe.

About Google Book Search

Google's mission is to organize the world's information and to make it universally accessible and useful. Google Book Search helps readers discover the world's books while helping authors and publishers reach new audiences. You can search through the full text of this book on the web at `http://books.google.com/`



Transferred to
GENERAL LIBRARY

TA
545
.B17

1752 1954

A RUDIMENTARY TREATISE

ON

LAND AND ENGINEERING SURVEYING,

WITH ALL THE MODERN IMPROVEMENTS

ARRANGED FOR

THE USE OF SCHOOLS AND PRIVATE STUDENTS,

ALSO FOR

PRACTICAL LAND SURVEYORS AND ENGINEERS.

IN TWO PARTS.

With Illustrations.

BY

T. hcmas BAKER, C.E.,

Author of "Levelling and Theodolite Surveying," in the Ninth Edition of Nesbit's Surveying, "Railway Engineering," "Integration of Differentials," &c., &c.

London:

JOHN WEALE,

ARCHITECTURAL LIBRARY, 59, HIGH HOLBORN.

MDCCCL.

LONDON:
PRINTED BY J. S. HODSON, 22, PORTUGAL STREET,
LINCOLN'S INN.

CONTENTS.

PART I.

LAND SURVEYING.

PAGE

Introduction vii

CHAPTER I.

Practical Geometry—Definitions 1
Problems in Practical Geometry 4

CHAPTER II.

The Chain, Offset-staff, and Cross 9
Directions for measuring lines on the ground 10
The common Drawing Compasses 11
The Hair Compasses . . . . . *ibid*
Plotting Scales 12
The Protractor 13
Planning Surveys 14
The Field Book . . . . . *ibid*
To Survey with the Chain and Cross 15
Tables of Linear and Square Measure . . . . . *ibid*
Problems I. to VII. on Surveying by the Chain and Cross . . . *ibid*

CHAPTER III.

Land Surveying by the Chain only 32
Preliminary Problems—Prob. I. To erect a Perpendicular with the Chain *ibid*
Prob. II. and III. To measure lines obstructed by ponds, buildings, &c. . 33
Prob. IV. To find the width of a river, which is too wide to be reached across by the chain (a new method) 35
Prob. V. Triangular Fields.—Methods of measuring and computing their areas.—Method of casting, &c. . . . . *ibid*
Prob. VI. Four sided Fields 40
Prob. VII. Fields having more than four sides.—Several examples of expeditiously measuring Fields of this kind.—Defects of old methods 42
Prob. VIII. To survey Woods, Lakes, &c. 48
Prob. IX. To survey a small estate, divided internally by straight fences, with a new and concise method of proving the true position of straight fences without measuring their lengths 50
The method of measuring hilly ground 52
The use of the Parallel Ruler in reducing crooked fences to straight ones 54
Surveying large estates or parishes by the Chain only 58

CHAPTER IV.

PAGE
The Vernier Scale 62
Plotting extensive surveys from this Scale 63
The Beam Compasses 64
The Pantagraph, for copying maps 65
Other methods of copying maps 67
The Prismatic Compass 69
The Box Sextant 71
The Optical Square 74
The Theodolite . . . . . *ibid*
The Circular Protractor 79
Planning extensive surveys—Improved form of the Field Book . . 80
Mr. Rodham's improved form of the Field Book, with Plan . . . 81

CHAPTER V.

Extensive Surveys of various kinds, either with or without the Theodolite 87
Railway Surveys 88
Engineering and other Surveys by the Theodolite 89
To survey Woods, Lakes, Harbours, &c., by the Theodolite. Prob. I. . 90
To survey Roads and Rivers. Prob. II. 93
To survey a part or the whole of a City or large Town. Prob. III. . 95
To determine the positions of several distant Objects or Stations. Prob. IV. 97
To survey a District where obstructions occur, for a Railway, &c. . . 98

CHAPTER VI.

SECTION I. The various methods of laying out any given Quantity of Land in any proposed regular or irregular Form, &c. Twelve Problems with original formulæ 99
SECTION II. On dividing Land of either equal or variable Value among various claimants, in proportion to their several claims. Six Problems, with original methods and formulæ 111
Roads, Quarries, Watering Places, &c., required to be set out previous to inclosing Commons, Wastes, &c. 122
The method of dividing and allotting the remaining part of the Common 123

PART II.

ENGINEERING SURVEYING.

CHAPTER I.

LEVELLING.

Definition of Levelling 125
Levelling Instruments . . . . . *ibid*
The Y Level . . . . . *ibid*
Troughton's Level 127
Gravatt's Level 128

PAGE
Water Level 129
Levelling Staves 130
Correction for Curvature . . . . . *ibid*
Principles and Practice of Levelling 132
To draw a sectional line on the earth's surface, the Levels of which have been taken 134
Level Book 135
Datum Line . . . . . *ibid*
Practical Level Book 136
Running a Check Level 137
Level Book for ditto 141
Levelling for the Formation of a Section 143
Level Book for Plotting the Section 144
To draw the Section 146
Cross Levels 147
Cross Levels on Road, &c. . . . . *ibid*
Parliamentary Plan and Section 148
Working Section . . . . . *ibid*
Level Book for Working Section 149
The Method of laying out Gradients 152
Rate of Inclination of Gradients 154
Tunnels . . . . . *ibid*
Levelling with the Theodolite . . . . . *ibid*

CHAPTER II.

The Method of laying out Railway Curves on the Ground.

(1.) On Railway Curves in general 156
(2.) On the different Methods of laying out Railway Curves . . . *ibid*
(3.) On Curve Rulers or Mechanical Railway Curves . . . . *ibid*
(4.) The Limit of Railway Curves 157
Prob. I. To determine the Radius of a Railway Curve. Cases I. to IV. *ibid*
Prob. II. To lay out a Railway Curve on the Ground by the Common Method. Cases I. and II. 159
Prob. III. To lay out a Railway Curve by Offsets from its Tangents. Cases I. to III. 163
Prob. IV. To lay out a Railway Curve by two Theodolites . . . 165
Prob. V. To lay out a Railway Curve by Offsets from its Chord or Chords 167
Prob. VI. To find the Radii of the compound Curve 168
Prob. VII. To fine one of the two Radii of the compound Curve, the other radius and tangent points being given 169
Prob. VIII. To find the Radii of the serpentine Curve mechanically . *ibid*
Prob. IX. To find one of the Radii of the serpentine Curve by calculation, the other radius and tangent points being given . . 170
Prob. X. To make a given deviation from a straight line of Railway by three Curves, &c. 171
Examples of expensive severance of property by improperly setting out Railway Curves . . . . . *ibid*
The practice of Engineers in the adoption of Curves in various Railways 172
Remarks on the invention of the Methods of laying out Railway Curves on the Ground 174

CHAPTER III.

PAGE
On setting out the surface widths of Railways 177
Problems I. to IV. On setting out the surface widths on all varieties of ground . . . . . *ibid*
Level Book 182
Prob. V. To calculate the quantity of land for a projected railway . 183
Prob. VI. To find the exact quantity of land for a railway . . . *ibid*

CHAPTER IV.

Tunnelling 184
Articles 1 to 6. On setting out Tunnels . . . . . *ibid*
Notes 1 to 6 on excavating Tunnels 186

CHAPTER V.

The methods of finding the contents of railway cuttings, &c. . . . 188
Tables for finding the contents of ditto . . . . . *ibid*
Prob. I. To find the contents of railway cuttings when the surface lines of the several sections are level 190
Prob. II. To find the contents from sectional areas 192
Prob. III. To adapt the General Table to all widths of formation level and ratios of slope 194
Prob. IV. To find the content of a cutting when each of the sides have two different ratios of slope . . . . . *ibid*
Prob. V. To find the content when the ratio of the slopes of the two sides are different 195
Prob. VI. To find the quantity of the cutting of a tunnel . . . *ibid*
Examples showing the errors of the methods of Mr. Bashforth and others *ibid*
Remarks on Mr. Bashforth's and the Author's method of finding the contents of cuttings 198

CHAPTER VI.

Problems and Formulæ of utility in Land and Engineering Surveying . 200
——— original and select, for the exercise of students 204
Centrifugal force of trains in railway curves 207
Formulæ for the super-elevation of the exterior rail 209

APPENDIX.

Tubular and other iron girder bridges 211
Brick and stone viaducts 214
Oblique or skew bridges 215
High level bridge, Newcastle upon Tyne . . . . . *ibid*
Wooden bridges 216
Table No. 1. Offsets to railway curves 217
—— No. 2. Correction of levels for parallax and refraction . . 218

ERRATA.

Page 15, line 18, for *lunar* read *linear*.
—— 37, —— 1, for *sum* read *some*.
—— 43, —— 15, for *commissions* read *commissioners*.
—— 43, —— 7 from bottom, for E read A.
—— 120, in formula for $4(\alpha+\beta)\,l\,V$ read $4(\alpha m+\beta o)\,l\,V$.
—— 120, first line in Note 1., for *formulæ* read *formula*.
—— 178, line 6, for a M read a B.
—— 180, —— 2, for M C′ read M D′.

INTRODUCTION.

LAND SURVEYING is a branch of the mathematics, applied to practical purposes, and had its origin, it is said, in Egypt, more than 1000 years before the Christian era; where the annual inundations of the Nile, and the consequent large deposits of mud, destroyed the land marks of the different proprietors. It, therefore, became necessary to determine these land marks by measurement, or to lay out the proper quantities of land claimed by the several proprietors, irrespective of their land marks thus destroyed. Hence the origin of the science of *geometry*, so called from its being compounded of two Greek words, signifying "*to measure the earth.*" This science, it is well known, is the foundation of the splendid system of mathematics, which has imparted to mankind a knowledge of the extent of the farthest visible universe, and of the laws by which ALMIGHTY POWER directs the celestial motions. But, notwithstanding the early origin of this science, in as far as it is applied to land surveying, it has received comparatively very little improvement almost up to the present time. The writers of extensive works on this subject being chiefly practical men, unacquainted with modern analysis, and some of them even ignorant of geometry, have successively produced works, which are little more than mere copies of those previously published, their examples being in all cases, simple and profuse without variety or elegance, the invariable result of such labours being a large volume, immeasurably behind the requirements of the subject as well as the science of modern times.

In the following work, I trust, it will be found that the author has avoided such profusion and want of variety in the subjects proposed, making, at the same time, the most important parts sufficiently clear to those students who have not had an opportunity of studying geometry and the higher branches of mathematics,—the demonstrations of all rules and formulæ not previously published, being given in notes, apart from the practical matter; while the demonstrations of all rules previously given in various works of geometry and analytical trigonometry, are omitted, to avoid increasing the size of the work, which is divided into *Two Parts*, viz., *Land and Engineering Surveying*, and each part into six Chapters; on each of which it will be proper to make a few observations.

CHAPTERS I. and II., on practical geometry and surveying by the chain and cross, are of two simple a character to admit of improvement, the author's object in these being condensation and clearness, with a sufficiency of examples to introduce the subject.

CHAPTER III., on surveying with the chain only, contains several new Problems, among which may be named Problem IV., which furnishes an original and concise method of finding the width of a large river. Problem VII. gives three methods of surveying fields of from five to seven sides, with only five chain lines, with examples of the numerous lines adopted in old methods; and Problem IX. gives a method of surveying a small estate of six fields by either five or four chain lines, with the method of proving the positions of straight fences; which positions all previous authors have determined by the crossing of two chain lines, or by prolonging each straight fence to two chain lines; which methods constitute no check on wrong entries in the Field Book. This Chapter concludes with several specimens of laying out main lines of extensive surveys, which occurred in the author's practice, including the main lines recommended by the Tithe Commissioners.

In CHAPTER IV. are given engravings and descriptions of the most efficient drawing and surveying instruments, which are chiefly taken from *Heather's Treatise on Mathematical Instruments*, in the Rudimentary Series, the best work on this subject. In this Chapter Rodham's method of keeping the Field Book, invented about fifty years ago, is given with slight alteration, on a folding plate, and the plan of an estate, to which the field book refers.

CHAPTER V. contains several surveys chiefly by the Theodolite, including surveys for railways and other engineering purposes.

CHAPTER VI., on dividing and inclosing land, commons, &c., has been almost entirely remodelled by the author, several formulæ, never before published, being given, for the expeditious laying out and dividing land of uniform or variable value, the

demonstrations of which are given in the notes, or among the Formulæ in Part II Chapter VI. This Chapter concludes Part I.; which, it will be seen treats exclu sively on land surveying.

PART II. of this work, may with propriety be called modern, if we excep Chapter 1. on levelling, which has been practised above a century, firstly for canals and secondly for railways; however, no good treatise on this subject appeared til that by *Mr. Simms*, whose plates and examples, being in the publisher's possession were adopted by the author in this work, as well as some parts of Mr. Simms' ac companying explanations of the subject. In this Chapter the author has also availed himself of *Heather's Treatise on Mathematical Instruments* for the plates of th levels, &c. and some parts of their descriptions.

CHAPTER II. treats of the various methods required for laying out railway curves in the ground; these methods were invented by the author about twenty-seven years ago, were first published in the *Gentleman's Diary*, secondly in the *Ninth Edition o Nesbits' Surveying*, thirdly in the author's *Railway Engineering*, and now for the fourth time in this work. The author trusts that it will be found, that he has treated this subject in an improved and truly practical manner: the investigations of the additional formulæ here required are given in the notes, the others will be found in the author's works above referred to, the geometrical constructions of compound and serpentine curves being here omitted, as being seldom used by practical men. *See Remarks on the Invention of the Method of laying out Railway Curves*, page 174.

In CHAPTER III. the methods of setting out the widths of railway cuttings, on all varieties of sloping and undulating ground, are carried out chiefly by original formulæ which are also given with their investigation, in the author's *Railway Engineering Mr. Simms* also wrote on this subject, in his work on *Levelling*, but without giving any formulæ; these methods may therefore be considered as entirely new modelled.

CHAPTER IV. is on tunnelling, on the setting out of which very little has been written by any author. The author has, he trusts, given clear and practical methods for this purpose, whether the tunnel be straight or curved, with copious notes, chiefly extracted from the *Practical Railway Engineer*, published by Mr. Weale in 4to., on the methods of excavating tunnels, &c.

CHAPTER V. is on the author's concise and original method of finding the contents of the earthwork of railways, almost entirely without calculation. This method was invented by the author about twelve years ago, *although another author, the errors of whose methods approximate to* 50 *per cent. as their maximum, has the assurance to claim the same methods*, (see remarks on this subject, page 198,) the errors of that author's method being further pointed out in this work, its falacy having been before fully established by rigid mathematical demonstration in the author's *Railway Engineering*.

CHAPTER VI. contains a collection of problems and formulæ of utility in land and engineering surveying, the investigation of which are either given or referred to in other works, with a collection of unsolved problems original and select, for the exercise of students. At the end of this Chapter is given *Pambour's Formulæ* for the super-elevation of the exterior rail in railway curves.

The work concludes with an Appendix, in which are given the dimensions of the famous tubular bridges, and several of the principal railway viaducts of various constructions which the late general extension of railways has called into existence in this kingdom.

From the preceding analysis of the contents of this work, as well as from an inspection of its several details, and a comparison of these with the works of other authors, it will at once be seen that of the twelve Chapters of which this work is composed three may be safely claimed by the author as having been originally drawn up by him, viz., Chapters II., III. and V. of Part II., while in all the other Chapters, excepting the first and second in Part I., considerable additions and improvements have also been made; and these in the compass of a smaller volume than those that treat only of land surveying. I trust, therefore, the intelligent reader will find that the author has placed this subject in the position it ought to occupy among the other scientific works of the age.

LAND AND ENGINEERING SURVEYING.

PART I.

LAND SURVEYING.

CHAPTER I.

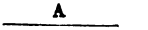
PREVIOUS to commencing the various subjects of Land and Engineering Surveying, it will be necessary to give a clear view of Practical Geometry, which is especially requisite for those who are unacquainted with this branch, as well as those parts of the mathematics which are equivalent to it.

PRACTICAL GEOMETRY.

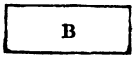
DEFINITIONS.

1. *A point* has no dimensions, neither length, breadth, nor thickness.

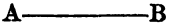
2. *A line* has length only, as A.



3. *A surface or plane* has length and breadth, as B.

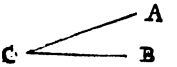


4. *A right or straight line* lies wholly in the same direction, as A B.

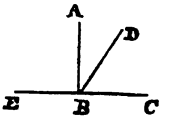


5. *Parallel lines* are always at the same distance, and never meet when prolonged, as A B and C D.

6. *An angle* is formed by the meeting of two lines, as A C, C B. It is called the angle A C B, the letter at the angular point C being read in the middle.



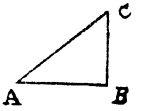
7. *A right angle* is formed by one right line standing erect or perpendicular to another; thus, A B C is a right angle, as is also A B E.



8. *An acute angle* is less than a right angle, as D B C.

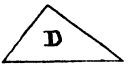
9. *An obtuse angle* is greater than a right angle, as D B E.

10. *A plane triangle* is a space included by three right lines, and has three angles.



11. *A right angled triangle* has one right angle, as A B C. The side A C, opposite the right angle, is called the hypothenuse; the sides A B and B C are respectively called the base and perpendicular.

12. *An obtuse angled triangle* has one obtuse angle, as the angle at B.



13. *An acute angled triangle* has all its three angles acute, as D.

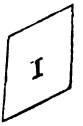
14. *An equilateral triangle* has three equal sides, and three equal angles, as E.

15. *An isosceles triangle* has two equal sides, and the third side greater or less than each of the equal sides, as F.

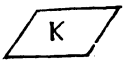
16. *A quadrilateral figure* is a space bounded by four right lines, and has four angles; when its opposite sides are equal, it is called a *parallelogram.*

17. *A square* has all its sides equal, and all its angles right angles, as G.

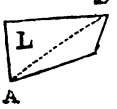
18. *A rectangle* is a right angled parallelogram, whose length exceeds its breadth, as B, (see figure to definition 2).



19. *A rhombus* is a parallelogram having all its sides and each pair of its opposite angles equal, as I.



20. *A rhomboid* is a parallelogram having its opposite sides and angles equal, as K.

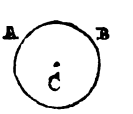


21. *A trapezium* is bounded by four straight lines, no two of which are parallel to each other, as L. A line connecting any two of its angles is called the *diagonal*, as A B.

22. *A trapezoid* is a quadrilateral, having two of its opposite sides parallel, and the remaining two not, as M.

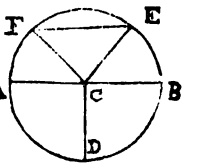
23. *Polygons* have more than four sides, and receive particular names, according to the number of their sides. Thus, a *pentagon* has five sides; a *hexagon*, six; a *heptagon*, seven; an *octagon*, eight; &c. They are called regular polygons, when all their sides and angles are equal, otherwise irregular polygons.

24. *A circle* is a plain figure, bounded by a curve line, called the circumference, which is everywhere equidistant from a point C within, called the centre.



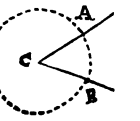
25. *An arc of a circle* is a part of the circumference, as A B.

26. The *diameter of a circle* is a straight line A B, passing through the centre C, and dividing the circle into two equal parts, each of which is called a *semicircle*. Half the diameter A C or C B is called the *radius*. If a radius C B be drawn at right angles to A B, it will divide the semicircle into two equal parts, each of which is called a *quadrant*, or one fourth of a circle. A *chord* is a right line joining the extremities of an arc, as F E. It divides the circle into two unequal parts called *segments*. If the radii C F, C E be drawn, the space, bounded by these radii and the arc F E, will be the *sector of a circle*.



27. The circumference of every circle is supposed to be divided into 360 equal parts, called degrees, and each degree into 60 minutes, each minute into 60 seconds, &c. Hence a semicircle contains 180 degrees, and a quadrant 90 degrees.

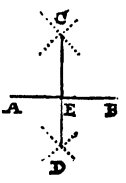
28. *The measure of an angle* is an arc of any circle, contained between the two lines which form the angle, the angular point being the centre; and it is estimated by the number of degrees contained in that arc:— thus the arc A B, the centre of which is C, is the measure of the angle A C B. If the angle A C B contain 42 degrees, 29 minutes, and 48 seconds, it is thus written 42° 29′ 48″.



PROBLEMS IN PRACTICAL GEOMETRY.

(In solving the five following problems only a pair of common compasses and a straight edge are required; the problems beyond the fifth require a scale of equal parts; and the two last a line of chords: all of which will be found in a common case of instruments.)

Problem I.

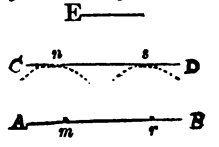


To divide a given straight line A B *into two equal parts.*

From the centres A and B, with any radius, or opening of the compasses, greater than half A B, describe two arcs, cutting each other in C and D; draw C D, and it will cut A B in the middle point E.

Problem II.

At a given distance E, *to draw a straight line* C D, *parallel to a given straight line* A B.

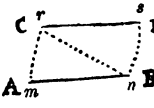


From any two points *m* and *r*, in the line A B, with a distance equal to E, describe the arcs *n* and *s*:—draw C D to touch these arcs, without cutting them, and it will be the parallel required.

Note. This problem, as well as the following one, is usually performed by an instrument called *the parallel ruler.*

Problem III.

Through a given point r, to draw a straight line C D *parallel to a given straight line* A B.

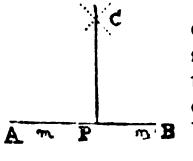


From any point *n* in the line A B, with the distance *n r*, describe the arc *r m*:—from centre *r*, with the same radius, describe the arc *n s*:—take the arc *m r* in the compasses, and apply it from *n* to *s*:—through *r* and *s* draw C D, which is the parallel required.

Problem IV.

From a given point P *in a straight line* A B *to erect a perpendicular.*

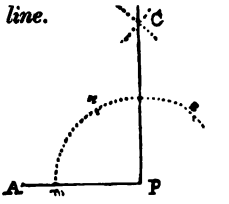
1. *When the point is in or near the middle of the line.*



On each side of the point P take any two equal distances, P *m*, P *n*; from the points *m* and *n*, as centres, with any radius greater than P *m*, describe two arcs cutting each other in C; through C, draw C P, and it will be the perpendicular required.

2. *When the point* P *is at the end of the line.*

With the centre P, and any radius, describe the arc *m r s*;—from the point *m*, with the same radius, turn the compasses twice on the arc, as at *r* and *s*:—again, with centres *r* and *s*, describe ares intersecting in C:—draw C P, and it will be the perpendicular required.



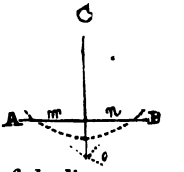
NOTE. This problem and the following one are usually done with an instrument called *the square*.

PROBLEM V.

From a given point C *to let fall a perpendicular to a given line.*

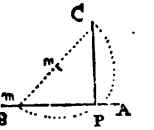
1. *When the point is nearly opposite the middle of the line.*

From C, as a centre, describe an arc to cut A B in *m* and *n*;—with centres *m* and *n*, and the same or any other radius, describe arcs intersecting in *o*: through C and *o* draw C *o*, the perpendicular required.



2. *When the point is nearly opposite the end of the line.*

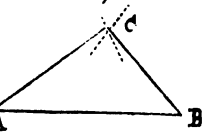
From C draw any line C *m* to meet B A, in any point *m*;—bisect C *m* in *n*, and with the centre *n*, and radius C *n*, or *m n*, describe an arc cutting B A in P. Draw C P for the perpendicular required.



PROBLEM VI.

To construct a triangle with three given right lines, any two of which must be greater than the third. (Euc. I. 22.)

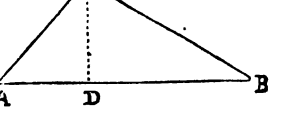
Let the three given lines be 5, 4 and 3 yards. From any scale of equal parts lay off the base A B = 5 yards; with the centre A and radius A C = 4 yards, describe an arc; with centre B and radius C B = 3 yards; describe another arc cutting the former arc in C:—draw A C and C B; then A B C is the triangle required.



PROBLEM VII.

Given the base and perpendicular, with the place of the latter on the base, to construct the triangle.

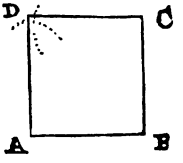
Let the base A B = 7, the perpendicular C D = 3, and the distance A D = 2 chains. Make A B = 7 and A D = 2;—at D erect the



perpendicular D C, which make = 3:—draw A C and C B; then A B C is the triangle required.

Problem VIII.

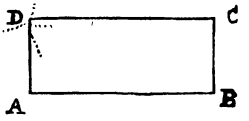
To describe a square, whose side shall be of a given length.



Let the given line A B be three chains. At the end B of the given line erect the perpendicular B C, (by Prob. IV. 2.) which make = A B:—with A and C as centres, and radius A B, describe arcs cutting each other in D: draw A D, D C, and the square will be completed.

Problem IX.

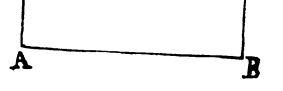
To describe a rectangled parallelogram having a given length and breadth.



Let the length A B = 5 chains, and the breadth B C = 2. At B erect the perpendicular B C, and make it = 2:—with the centre A and radius B C describe an arc; and with centre C and radius A B, describe another arc, cutting the former in D:—join A D, D C to complete the rectangle.

Problem X.

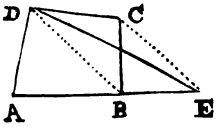
The base and two perpendiculars being given to construct a trapezoid.



Let the base A B = 6, and the perpendiculars A D and B C, 2 and 3 chains respectively. Draw the perpendiculars A D, D C, as given above, and join D C, thus completing the trapezoid.

Problem XI.

To make a triangle equal to a given trapezium A B C D.



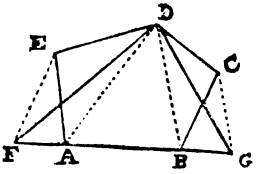
Draw the diagonal D B, and C E parallel to it, meeting A B prolonged in E:—join D E; then shall the triangle A D E be equal to the trapezium A B C D.

Problem XII.

To make a triangle equal to the figure A B C D E A.

Draw the diagonals D A, D B, and the lines E F, C G,

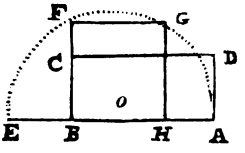
parallel to them, meeting the base A B, produced both ways, in F and G :—join D F, D G; then the triangle D F G will be equal to the given figure A B C D E.



Problem XIII.

To make a square equal to a given rectangle A B C D.

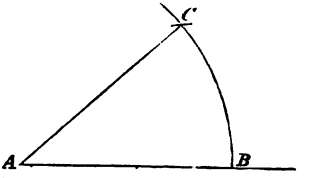
Produce one side A B till B E be equal to B C:—bisect A E in *o*; on which as a centre, with radius A *o*, describe a semicircle, and prolong B C to meet it in F:—on B F describe the square B F G H, and it will be equal to the rectangle A B C D, as required.



Problem XIV.

To set off an angle to contain a given number of degrees.

Let the angle be required to contain 41 degrees. Open the compasses to the extent of 60° upon the line of chords, and, setting one foot upon A, with this extent, describe an arc cutting A B in B; then taking the extent of 41° from the same line of chords, set it off from B to C; join A C; then B A C is the angle required.



Problem XV.

To measure an angle contained by two straight lines.
(See last figure.)

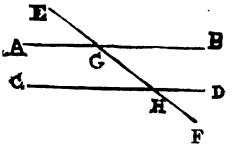
Let A B, A C contain the angle to be measured. Open the compasses to the extent of 60°, as before, on the line of chords, and with this radius describe the arc B C, cutting A B, A C produced, if necessary, in B and C; then extend the compasses from B to C, and this extent, applied to the line of chords, will reach to 41°, the required measure of the angle B A C.

A right angle, or perpendicular, may be laid off by extending the arc B C, and setting off the extent of 90° thereon. Also an angle greater than 90° may be laid off, by still further extending the arc, and laying the excess of the arc above 90°, from the end of the 90th degree.

Note. Angles are more correctly and expeditiously laid off and measured by an instrument called the protractor, to be hereafter described.

GEOMETRICAL THEOREMS.

(Necessary to be known by Surveyors).



THEOREM I.

Angles vertically opposite are equal:—thus the angle A G E = angle H G B, and E G B = A G H. (Euc. I. 15.)

THEOREM II.

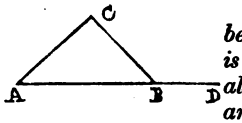
(See last figure.)

A right line E F, *cutting two parallel right lines* A B, C D, *makes the alternate angle equal, &c:*—thus the angles A G H, G H D are equal; also the exterior angle E G B is equal to the interior and opposite G H D. (Euc. I. 29.)

THEOREM III.

The greatest side of every triangle is opposite the greatest angle (Euc. I. 18.)

THEOREM IV.



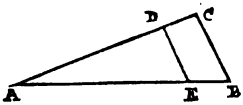
Let the side A B *of the triangle* A B C *be produced to* D, *the exterior angle* C B D *is equal to the interior angles at* A *and* C; *also the three interior angles of the triangle are equal to two right angles.* (Euc. I. 32.) Whence any two angles of a triangle being given the third becomes known.

THEOREM V.

(See figure to Definition 11.)

Let A B C be a right angled triangle, having a right angle at B; then, *the square on the side* A C *is equal to the sum of the square on the sides* A B, B C. (Euc. I. 47.) Whence any two sides of a right angled triangle being given the third becomes known.

THEOREM VI.



In any triangle A B C, let D E be drawn parallel to one of its sides, C B; then, A B *is to* A E *as* B C *is to* D E; and the triangles are said to be similar. (Euc. VI. 2.)

THEOREM VII.

(See last figure.)

Let A B C, A E D be similar triangles; then, *the triangle* A B C *is to the triangle* A E D *as the square* A B *is to the square of* A E: that is, similar triangles are to one another in the duplicate ratio of their homologous sides. (Euc. VI. 19.)

THEOREM VIII.

All similar figures are to one another as the squares of their homologous, or like, sides. (Euc. III. 20.)

THEOREM IX.

All similar solids are to one another as the cubes of their like linear dimensions. (Euc. VI. 24.)

CHAPTER II.

DESCRIPTION OF INSTRUMENTS USED FOR MEASURING AND PLANNING SMALL SURVEYS.

THE CHAIN.

THE chain, usually called Gunter's chain, is almost generally used in the British dominions, for measuring the distances required in a survey. It is 66 feet, or 4 poles, in length, and is divided into 100 links, which are joined by rings. The length of each link, together with half the rings connecting it with the adjoining links, is consequently $\frac{66}{100}$ of a foot, or $\frac{66 \times 12}{100} = 7{\cdot}92$ inches. At every tenth link from each end is attached a piece of brass with notches; that at the tenth link has one notch, that at the 20th two notches, that at the 30th three, that at the 40th four, the middle of the chain, or the 50th link being marked with a large round piece of brass; hence, any distance on the chain may be readily counted. Part of the first link, at each end, is formed into a large ring for the purpose of holding it with the hand.

The chain acquires extension by much use, it should, therefore, be frequently examined, and adjusted to the proper length by taking out some of the rings between the links: for this purpose, chains having three rings between each link are to be preferred to those having only two.

THE OFFSET STAFF.

The offset staff is used to measure short distances, called offsets; hence its name. It is usually ten links in length, the links being numbered thereon with the figs. 1, 2, 3, &c. It is usually pointed with iron at one end, for the purpose of fixing it in the ground, as an object for ranging lines, for marking stations, &c.

THE CROSS.

The cross is an instrument used by surveyors to erect perpendiculars. It is usually a round piece of sycamore, box, or

mahogany, about four inches in diameter, with two folding sights at right angles to each other, or more commonly with two fine grooves sawed at right angles to each other, which answer the purpose of sights. It is sometimes fixed on a staff, of convenient length for use, pointed with iron at the bottom, that it may be fixed firmly in the ground: but it is found more commodious in practice to have a small pocket cross, which may be readily fitted to the offset-staff, either by an iron spike on the cross being inserted in a hole made in the offset-staff, or the offset-staff being passed through a hole made in the cross, to about the eighth link from the piked end, at which place the staff must be shouldered, that the cross may rest firmly.

DIRECTIONS FOR MEASURING LINES ON THE GROUND.

Besides the instruments already described, ten arrows must be provided, about 12 inches long, pointed at the end, so as to be readily pressed into the ground, and turned at the other end, so as to form a ring to serve for a handle.

In using the chain, marks are to be set up at the extremities of the line to be measured, as well as at its intermediate points, if its extremities cannot be seen from one another, on account of hills, woods, hedges, or other obstructions. Two persons are then required by the surveyor to perform the measurement. The chain leader starts with the ten arrows in his left hand, and one end of the chain in his right; while the follower remains at the starting point, who, looking at the staff or staves, that mark the line to be measured, directs the leader to extend the chain in the direction of the staff or staves. The leader then puts down one of his arrows, and proceeds a second chain's length in the same direction, while the follower comes up to the arrow first put down. A second arrow being now put down by the leader, the first is taken up by the follower; and the same operation is repeated till the leader has expended all his arrows. Ten chains, or 1000 links, having now been measured and noted in the field book, the follower returns the ten arrows to the leader, and the same operation is repeated as often as necessary. When the leader arrives at the end of the line, the number of arrows in the follower's hand shews the number of chains measured since the last exchange of arrows noted in the field book, and the number of links extending from the last arrow to the mark or staff at the extremity of the line, being also added, gives the entire measurement of the line. Thus, if the arrows have been exchanged seven times, and

	7000
	600
	83
	7683

if the follower have six arrows, and from the arrow last put down to the end of the line be 83 links, the whole measurement will be 7683 links, or 76 chains 83 links, which is usually written thus—76·83 chains, the two last figures being decimals of a chain.

In using the chain, care must be taken to stretch it always with the same tension, as it will extend by much use, and will therefore require to be examined occasionally, and shortened, if necessary. But a good chain may be used several days, on tolerably smooth ground, without any material extension.

The surveyor must mark, or caused to be marked, every station on the line, while it is being measured, with a staff or cross on the ground, entering its distance in the field book.

When a survey is made for a finished plan, all remarkable objects should be noted down; as buildings, roads, rivers, ponds, footpaths, gates, &c.

The boundary of the estate measured ought to be carefully observed. If the ditch be outside the boundary fence, it usually belongs to the estate, and *vice versa;* although this is not uniformly the case; therefore, inquiry ought to be made with respect to the real boundary.

In some places five links from the hedge-posts or roots of the quickwood are allowed for the breadth of the ditch, but this breadth varies to as far as even ten links, especially in swampy countries.

All ditches and fences must be measured with the fields to which they belong, when the full quantity on the plan is required: but when the growing crops only are to be measured, only so much as is occupied by the crops.

Instruments for Laying Down or Plotting Surveys.

THE COMMON DRAWING COMPASSES.

This instrument consists of two legs moveable about a joint, so that the points at the extremities of the legs may be set at any required distance from one another: it is used to transfer and measure distances, and to describe arcs and circles.

THE HAIR COMPASSES.

The hair compasses ought to be used where greater accuracy is required in transferring distances, than can be obtained by the set of the joint of the common compasses. In the hair compasses the upper part of one of the steel points is formed into a bent spring, which, being fastened at one extremity to the leg of the compasses almost close up to the joint, is held at

the other end by a screw. A groove is formed in the shank, which receives the spring when screwed up tight; and by turning the screw backwards the steel point may be gradually allowed to be pulled backwards by the spring, and may again be gradually pulled forwards by the screw being turned forwards.

Fig. 1. Fig. 2.

Fig. 3

Fig. 1, represents these compasses when shut; fig. 2, represents them open, with the screw turned backwards, and the steel point *p*, in consequence, moved backwards by its spring *s*, from the position represented by the dotted lines, which it would have when screwed tight up. Fig. 3, represents a key, of which the two points fit into the two holes seen in the nut *n* of the joint; and by turning this nut the joint is made stiffer or easier at pleasure.

To take a distance with the hair compasses.—Open them as nearly as you can to the required distance, set the fixed leg on the point from which the distance is to be taken, and make the extremity of the other leg coincide accurately with the other end of the required distance, by turning the screw.

NOTE. There are several other kinds of compasses, used for planning; as those with moveable points, for the introduction of black lead pencils or ink points, beam compasses for taking large distances, proportional compasses, &c., the uses of which are easily learned. (See *Heather's Treatise on Mathematical Instruments.*)

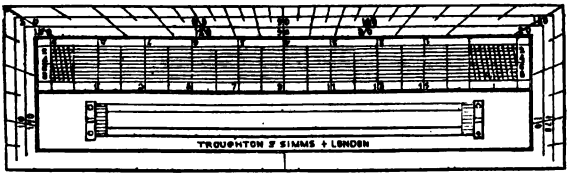
PLOTTING SCALES.

Plotting scales, also called feather-edged scales, are straight rulers, usually about 12 inches long. Each ruler has scales of equal parts, decimally divided, placed on its edges, which are made sloping, so that the extremities of the strokes marking the divisions lie close to the paper. The primary divisions represent chains, and the subdivisions ten links each, the intermediate links being determined by the eye. Plotting scales may be procured in sets, each with a different number of

chains to the inch. They are usually made of ivory or box, and each provided with a small scale called an offset scale for laying down the offsets. In using these scales, the first division or zero, on the plotting scale, is placed coincident with the beginning of the line to be plotted, and so as just to touch that line with the feather edge: the end of the offset scale is then placed in contact with the edge of the plotting scale; and thus the offsets may be expeditiously pricked off, for which purpose an instrument called a pricker is used, but a hard black lead pencil with a fine point, is greatly to be preferred, as it does not injure the paper.

THE PROTRACTOR.

The instrument represented in the annexed figure, is usually supplied with a pocket case of instruments. It is made of ivory, 6 inches long and $1\frac{3}{4}$ broad. On the face of the instrument, round three of its edges, which are feathered, the protractor is formed for readily setting off angles.



In using the protractor, the fourth edge, which is quite plain, with the exception of a single stroke in the middle, is to be made to coincide with the line, from which the angle is to be set off, and the stroke in the middle to coincide with the angular point in the line, at which the angle is to be set off; a mark is then to be made with a fine pointed black lead pencil, or with a pricking point, at the point on the paper which coincides with the stroke on the protractor marked with the number of degrees in the angle required to be drawn; and the protractor being now removed, a straight line is to be drawn from the angular point in the given line to the point thus marked off. The instrument has, on the same face, two diagonal scales, (which are now little used by surveyors,) and, on the opposite face, scales of equal parts, &c.

The vernier scale and circular protractor, the uses of which will be readily understood, are best adapted to laying down extensive surveys, where great accuracy is required.

Planning Surveys.

In planning or plotting surveys, the upper part of the paper or book, on which the plan is made, should always, if possible, be the north. The chain lines, buildings, fences, &c., ought first to be drawn with a fine black lead pencil: the first should then be dotted with ink, and the latter neatly drawn. Great care is required in the construction of the plan, when the dimensions are to be measured therefrom with the scale. The scale should never be more than three chains to an inch, for when the parts of a plan are large, the dimensions may be taken with greater accuracy. After having found the content of the field or fields, &c., of which any plan consists, it may be laid down by any scale to give it a more convenient size.

THE FIELD BOOK.

The method now generally adopted in setting down field notes, and which has long been found to be the best in practice, is to begin at the bottom of the page and write upwards.

Each page of the book is usually divided into three columns. The middle column is for distances measured on the chain line, at which hedges are crossed, or offsets, stations, or other marks are made; and in the right and left columns, those offsets, marks, and any other necessary observations thereon, must be entered, accordingly as they are situated on the right or left of the chain line.

The crossing of roads, rivers, hedges, &c., are, by some surveyors, shewn in the field book, by lines drawn across the middle column at the distances where they are crossed, and by others these crossings are shewn by lines drawn on part of the right and left hand columns, opposite the distances where they are crossed by the chain line; and buildings, turns of fences, corners of fields, to which offsets are taken, are usually shewn by lines sketched in a similar situation to the middle column, as the fences, buildings, &c., have to the chain line. Thus a representation of the chief objects in the survey may be sketched in the field, which will give essential assistance in laying down the plan. The stations are usually numbered, for the sake of reference, and marked thus ⊙. The bearing of the first main line is usually taken by surveyors, from which the position of the plan with respect to the north is determined. This may be done by a common pocket compass, where great accuracy is not required: but this will be more fully discussed in treating of surveys by the theodolite, further on.

R. of ⊙ 2, and L. of ⊙ 5, &c., denote that the following lines are measured to the right of station 2, and to the left of station 5, respectively.

To Survey with the Chain and Cross.

An acre of land is equal to 10 square chains, that is 10 chains in length and one in breadth, or 1000 links in length and 100 in breadth; an acre, therefore, contains 100,000 square links, as per table of square measure below. Hence the contents in square links are, in the following examples, divided by 100,000, or what is the same thing 5 figures to the right are cut off for decimals, the figures remaining on the left being acres. The decimals are then multiplied by 4 for rods, and again by 40 for poles.

The following tables exhibit the number of chains and links in the different units of lineal measure, and the number of square chains and links in the different units of square measure.

A TABLE OF LUNAR MEASURES.

Links.	Feet.	Yards.	Poles.			
25	16½	5½	1			
				Chains.		
100	66	22	4	1		
					Furlongs.	
1,000	660	220	40	10	1	
						Mile.
8,000	5,280	1,760	320	80	8	1

A TABLE OF SQUARE MEASURES.

Sq. Links.	Sq. Feet.	Sq. Yards.	Sq. Poles or Perches				
625	272¼	30¼	1				
				Sq.Chs.			
10,000	4,356	484	16	1			
					Rods.		
25,000	10,890	1,210	40	2½	1		
						Acres.	
100,000	43,560	4,840	160	10	4	1	
							Sq. Mile.
64,000,000	2,787,400	3,097,600	102,400	6,400	2,560	640	1

Problem I.

SQUARE AND RECTANGULAR FIELDS.

Square and rectangular fields seldom occur in the practice of the surveyor; small plots of ground of these forms, however, frequently present themselves.

Fix the cross in a corner of the square or rectangular piece

of ground, and if the sides be at right angles, measure one of them, and enter its length in the field book. This must be repeated at every side and angle; and if all the angles are found to be right and all the sides equal, the piece of ground is square; but, if only the opposite sides are equal, it is rectangular.

TO FIND THE CONTENT.

RULE. Multiply the length by the breadth in links, and the product will be the contents in square links, which reduce to acres, rods, and poles by the preceding table for square measure, as in the following examples.

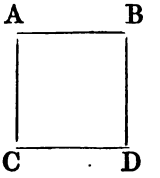
If the dimensions be given in yards, divide the product by 4840 for acres, &c.

EXAMPLES.

1. What is the area of the square A B C D, whose side is 478 links?

Here the length and breadth are equal, hence

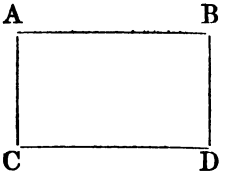
```
  478
  478
 ----
 3824
3346
1912
 ----
2·28484
    4
 ----
1·13936
   40
 ----
5·57440
```



Area 2*a*. 1*r*. 5½*p*.

2. The length of the rectangular field A B C D, is 2450, and its breadth 1226 links; required the plan and area.

```
  2450
  1226
 -----
 14700
 490
490
245
 -----
30·03700
    4
 -----
0·14800
   40
 -----
5·92000
```



Area 30*a*. 0*r*. 6*p*. nearly.

3. Required the area of a square, in acres, the side of which is 132 yards.

$$\frac{132 \times 132}{4840} = 3{\cdot}6 \text{ acres} = 3a.\ 2r.\ 16p. \text{ the area.}$$

4. What is the area of a rectangle whose length is 2470 links, and its breadth 1114 links?

$$2470 \times 1114 = 27{\cdot}51580 = 27a.\ 2r.\ 2\tfrac{1}{2}p. \text{ the area.}$$

5. The length of a rectangular field is 324 yards, and its breadth 235, what is its content in acres?

Ans. $15a.\ 2r.\ 37p.$

6. What is the area of a rectangular pleasure ground, the length of which is 960 and its breadth 125 links?

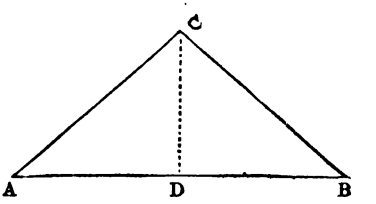
Ans. $1a.\ 0r.\ 32p.$

Problem II.

TRIANGULAR FIELDS.

1. Let A B C be a triangle, of which the survey, plan, and content are required.

Set up poles or marks at the angles A, B, and C, and measure from A towards B, and when at or near D, try with the cross for the place of the perpendicular C D; plant the cross, and turn it till the marks A and B can be seen through one of the grooves; then look through the other groove, and, if the mark at C can be seen through it, the cross is in the right place for the perpendicular; if not, move the cross backward or forward till the three marks can be seen as before directed. Suppose the distance A D to be 625 links, and the whole A B, to be 1257 links; return to D, and measure the perpendicular D C, which suppose to be 628 links, thus completing the survey of the triangle.



CONSTRUCTION.

From a scale of equal parts, or plotting scale, lay off the base A B, = 1257 links; on which take A D, = 625 links; at D erect the perpendicular D C, which make = 628 links; join A C, C B, then A B C is the plan of the triangle.

TO FIND THE CONTENT.

RULE. Mutiply the base by the perpendicular, and half the product will be the area.

EXAMPLES.

1. The dimensions being the same as found above, required the content.

Ans. $1257 \times 628 \div 2 = 3{\cdot}94698$ acres $= 3a.\ 3r.\ 32p.$

2. The distance from the beginning of the base to the place of the perpendicular is 375 links, the whole base 954, and the perpendicular 246; what is the area of the triangle.

$954 \times 246 \div 2 = 1{\cdot}17342 = 1a.\ 0r.\ 27\frac{1}{2}p.$ the content.

3. Measuring the base of a triangle the place of the perpendicular was found at 863 links, and its length 645; the whole base was 1434 links; required the plan and area.

Area. $4a.\ 2r.\ 20p.$

PROBLEM III.

FIELDS IN THE FORM OF TRAPEZIUMS.

Fields in this form are usually divided into two triangles by a diagonal, which is a base to both the triangles.

Let ABCD be a field in the form of a trapezium, the plan and area of which is required.

Measure from A towards C; and let the place of the perpendicular mB be at 5·52, and its length 3·76, also let the place of the perpendicular nD be at 11·82, and its length 3·44, and the length of the whole diagonal AC be 13·91 chains, which completes the survey: but it is usual also to measure the other diagonal BD for a proof line, which is found to be 9·56 chains.

NOTE 1. The construction of each of the two triangles, forming the trapezium, is the same as the construction given to the first example in PROB. II.

NOTE 2. The longer of the two diagonals should always be selected for the base of the two triangles, forming the trapezium, for sometimes the perpendicular will not fall on the shorter diagonal, without its being prolonged; and when this is the case with both diagonals, one of the sides may be taken for a base, or two of the sides, if necessary.

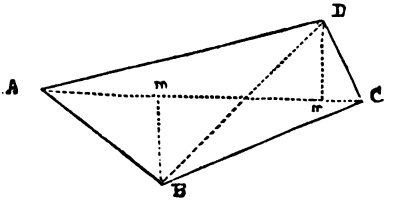
TO FIND THE CONTENT.

RULE. Multiply the sum of the two perpendiculars by the diagonal, and half the product will be the content.

EXAMPLES.

1. Let the measurement of a trapezium be as above found, required the content.

344
376
720
1391
27820
9737
2)10·01520
5·00760
4
0·03040
40
1·21600



Ans. 5*a*. 0*r*. 1*p*.

2. From the following notes, plan and find the content of a field.

Perpendiculars on left.	Base or Station Line.	Perpendiculars on right.
	to ⊙ C	
	3250	
	2504	1046 D
B 1278	1272	
Begin	at ⊙ A	and range West.

Content. 37*a*. 3*r*. 2*p*.

3. Give the plan and area of a field from the following notes.

	A C	
	872	
B 652	731	
	423	545 C
Begin at	⊙ A	and range E.

Area. 5*a*. 0*r*. 35*p*.

ANOTHER METHOD.

A four-sided field may frequently be surveyed by dividing it into two triangles and a trapezoid, by perpendiculars on its longest side.

TO FIND THE CONTENT.

RULE. Multiply the sum of the two perpendiculars by their distance on the base line for the double area. The double areas of the two triangles must be found as in PROB. II., and both be

added to the double area of the trapezoid; the sum being divided by 2, will give the content required.

EXAMPLES.

1. Required the survey and area of the following field.

Measure the base A B, and put down in the field book the distances of P and Q, where the perpendiculars rise, &c., as below.

	to ⊙ D.
	AB=1110
QD=595	AQ= 745
PC=352	AP= 110
Perpen.	From ⊙ A go E.

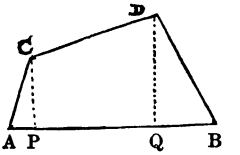
Trapezoid P C D Q.

352 } perp.
595 }

947 sum.
635 = PQ.

Triangle ACP.	Triangle QDB.
352	595
110	352
38720	9975
	3570
	1785
	217175

4735
2841
5682
601345
38720
217175
2)8·57240
4·2862
4
1·1448
40
5·7920



Area. 4*a*. 1*r*. 5$\frac{3}{4}$*p*.

2. Required the plans and areas of two fields from the following notes.

	A B	
	1169	
E	615	339 D.
G	234	461 C.
From	⊙ A go	W.

	A B	
	1448	
D 513	1102	E.
C 683	436	G.
From	⊙ A	go E.

Problem IV.

TO SURVEY FIELDS CONTAINED BY MORE THAN FOUR SIDES.

Fields or plots of ground bounded by more than four sides, may be surveyed by dividing them into trapeziums and triangles.—Thus, a field of five sides may be divided into a trapezium and a triangle; of six sides, into two trapeziums; of seven sides into two trapeziums and a triangle; &c.

TO FIND THE CONTENT.

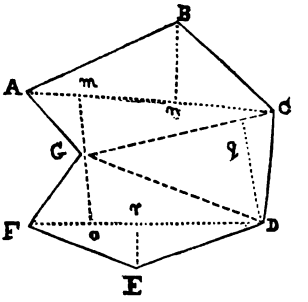
Rule. By the two last Problems, find the double areas of

each trapezium and triangle in the field; add all the double areas together, and half their sum will be the content.

EXAMPLES.

1. Lay down a field and find its area from the following dimensions.

	to ⊙ D	
	520	
p	288	80 E
G 120	206	*o*
	Go to ⊙ F.	
	to ⊙ G	
	440	
D 230	152	*q*
	L. of ⊙ C.	
	to ⊙ C	
	550	
B 180	410	*n*
m	135	130 G
Begin	at ⊙ A	range E.



CONSTRUCTION.

The above field is divided into two trapeziums A B C G, G D E F and the triangle G C D.—Draw the diagonal A C, which make = 550 links; at 135 and 410 set off the perpendiculars $mC = 130$, and $nB = 180$ links respectively; join A B, B C, C G, and G A, and the first trapezium will be completed. Then on C G, lay off $Cq = 152$, and draw the perpendicular $qD = 230$; join C D, D G, and the triangle is completed. Lastly, with centre G and radius $oG = 120$ describe an arc; and with centre D and radius $oD = 314$ (= 520 − 206) describe another arc, intersecting the former in *o*: through *o* draw the diagonal D F = 520 links, upon which, at 288 links, draw the perpendicular *p* E; join D E, E F, F G, and the figure will be completed.

130	440	120
180	230	80
310	13200	200
550	88	520
15500	101200	104000
1550		
170500		

Double areas.

170500 trap. A B C G.
101200 tri. C D G.
104000 trap. D E F G.

2)3·75700

1·87850 — 1*a.* 3*r.* 20½*p.*

2. Required the plans and areas of two fields from the following dimensions.

First Field.

	to ⊙ A	
	504	Base.
E 98	233	
Return	to ⊙ B	
	to ⊙ D	
	673	Diag.
C 207	472	
	427	268 B.
Begin	at ⊙ A	range W.

Area 1*a.* 3*r.* 15*p.*

Second Field.

	to ⊙ F	
	970	Diag.
	520	181 B.
E 290	413	
	R. of ⊙ D	
	to ⊙ D	
	744	Diag.
C 161	386	
	303	333 B.
Begin	at ⊙ A	rangeW.

Area 4*a.* 0*r.* 9½*p.*

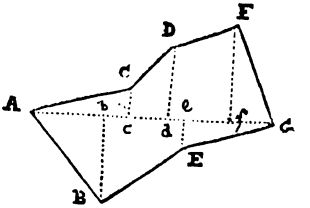
ANOTHER METHOD.

A small piece of land, having several sides, may sometimes be most conveniently measured by taking one diagonal, and upon it erecting perpendiculars to all the angles on each side of it. The piece will thus be divided into right angled triangles and trapezoids, the areas of which must be calculated as in the two last Problems.

EXAMPLES.

1. Required the plan and area of a field from the following notes.

	to ⊙ G	
	1020	
F 470	890	*f*
e	610	50 E.
D 320	585	*d*
C 70	440	*c*
b	315	350 B
Begin	at ⊙ A	go E.



NOTE. The method of planning the above field is sufficiently clear, from the preceding field-notes and from what has been already done.

Triangle A C *c*.		Trape. C *c d* D.		Trape. D *d f* F.		Tri. F *f* G.		Tri. A *b* B.	
A *c* =	440	D *d* =	320	D *d* =	320	*f* G =	130	A *b* =	315
C *c* =	70	C *c* =	70	F *f* =	470	F *f* =	470	B *b* =	350
	30800	sum =	390	sum =	790		9100		15750
		c d =	145	*e f* =	395		52		1545
			13050		3950		61100		170250
			435		2370				
			56550		240950				

Trapezoid B *b e* E.		Triangle E *e* G.	
B *b* =	350	G *e* =	410
E *e* =	50	E *e* =	50
sum =	400		20500
b e =	295		
	118000		

Double areas.

30800
56550
240950
61100
170250
118000
20500

2) 698150

3·49075

= 3*a*. 1*r*. 38½*p*. Area.

2. Lay down two pieces of ground, and find their areas, from the following dimensions.

First piece.

	to ⊙ R	
o	3401	*o*
I 579	2930	*i*
h	2736	730 H
g	2110	762 *b*
F 288	1972	*f*
e	1588	436 E
d	1030	740 D
C 500	888	*c*
b	300	550 B
o	000	*o*
Begin	at ⊙ A	go E.

Area 30*a*. 3*r*. 6*p*.

Second piece.

	to ⊙ L	
M 920	4300	*o*
k	3340	590 K
I 790	3060	*i*
h	2690	320 H
G 1340	2550	*g*
F 800	1760	*f*
e	1560	540 E
d	930	300 D
C 910	610	*c*
o	000	600 B
Begin	at ⊙ A	go W.

Area 53*a*. 2*r*. 31*p*.

Problem V.

FIELDS INCLUDED BY ANY NUMBER OF CROOKED OR CURVED SIDES.

When a field or estate is bounded by crooked fences, a line must be measured as near to each of them, as the angles or bends will permit; in doing which an offset must be taken to each corner or bend in the fence. When the fences are curved, these offsets must be taken so as neither to exclude nor include any of the land belonging to the ground to be measured. The offsets or perpendiculars thus erected, will divide the whole offset space into right angled triangles and trapezoids, the areas of which may be found as already shewn.

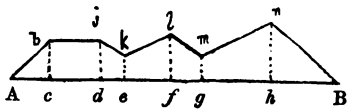
Note 1. When the offsets are short, that is, not greatly exceeding a chain in length, their places on the line may be found by laying the offset staff at right angles to the chain, as nearly as can be judged by the eye; but when the offsets are large, and correctness is required, their places must be found by the cross, and measured by the chain.

Note 2. The quickest method of laying down offsets, is, by laying the feather edge of the plotting scale against the base or chain line, and sliding the offset scale along the feather edge to the several distances of the offsets and pricking off their lengths, corresponding to their several distances.

Note 3. Unskilful surveyors usually add all the offsets taken on one line together and divide the sum by their number for a mean breadth; but this method is very erroneous, especially where the offsets vary greatly in length, and should therefore be avoided where great accuracy is required.

EXAMPLES.

1. Required the plan and content of a right-lined piece of ground by offsets, from the following notes.



	to ⊙ B	
o	955	
n 91	785	*h*
m 57	634	*g*
l 88	510	*f*
k 70	340	*e*
i 84	220	*d*
h 62	45	*c*
o	00	
Begin	at ⊙ A	range E.

A c = 45	$c h$ = 62	$d i$ = 84	$e k$ = 70	$f l$ = 88
$c h$ = 62	$d i$ = 84	$e k$ = 70	$f l$ = 88	$g m$ = 57
90	146	154	158	145
270	$c d$ = 175	$d e$ = 120	$e f$ = 170	$f g$ = 124
2790	730	18400	11060	580
	1022		158	290
	146			145
	25550		26860	17980

Double areas.

$g m$ = 57	h' B = 170	2790
$h' n$ = 91	$h' n$ = 91	25550
148	170	18480
$g h'$ = 151	1530	26860
148	15470	17980
740		22348
148		15470
22348		2)1·29478
		0·64739 = 0*a.* 2*r.* 23*p.*

Calculation by the erroneous method (See Note 3).

00
62
84
70
88
57
91
00
—
8)452
—
56½

955 = A B.
56½ = mean breadth.

5730
4775
477

0·53957 = 0*a.* 2*r.* 6*p.* Content by this method, *which is* 17 *perches too little.* For this method is always erroneous except when the offsets stand at equal distances from one another, and when the first and last offsets are both 0.

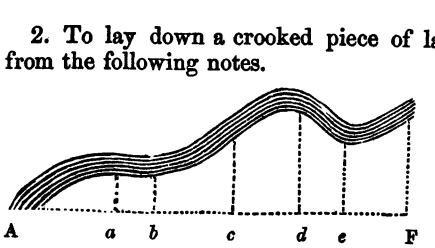
Some omit all the offsets that are 0, dividing the sum of the offsets by the number of real offsets; by this method we shall have

6)452
—
75½

955
75½
4775
6685
318

0·71947 = 0*a.* 2*r.* 34*p*, *which is* 11 *poles too much.*

2. To lay down a crooked piece of land, adjoining a river from the following notes.

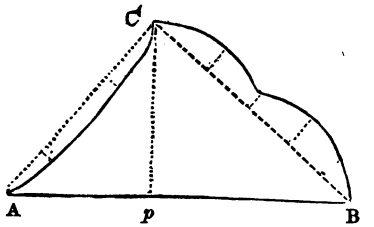


	to ⊙ F
197	720
163	600
226	500
139	350
80	200
74	100
0	000
Begin	at ⊙ A

[E. range

The content is found by the same method as in the preceding example.

3. Plan and find the area of a field from the subjoined notes.



	to ⊙ A
0	480
37	350
28	160
0	000
	L. of ⊙ C

	to ⊙ C	
	585	0
	450	57
	320	40
	200	72
	100	47
	000	0
	L. of ⊙ B	

	to ⊙ B
	743
C 382	290
From	⊙ A

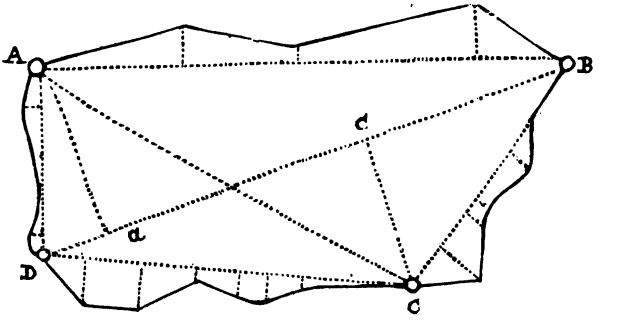
p go E.

Having found the area of the triangle A B C, the areas of the offsets on the line B C must be added thereto, and the sum of the areas of the *insets* on the line C A must be substracted from the sum, and the remainder will be the content of the field.

NOTE. The area of the triangle A B C may be found, when the measurement of all three sides are given, (which is the case in the present example,) either by calculation, as shall hereafter be shewn, or by measuring the perpendicular from the plan, which, as already shewn, may be laid down from the three sides of the main triangle.

4. The notes, plan, and content of the following field, the lines

for the measurement of which are sketched out as below, are required.



Here the double area of the trapezium A B C D is found, as in Problem III, to which the double areas of the several offsets on the lines A B, B C, C D, D A must be added, and the sum will be the double area of the whole.

NOTE. A proof line, as A C should be measured, that the accuracy of the work may be insured:—but often pieces of ground are measured with the chain and cross that are never laid down, in this case great care should be taken in entering the field notes, and in making the calculation.

5. Required the plan and area of a field from the following notes.

	To fence.	
80	2024	
84	1900	to ⊙ D
108	1200	
66	800	
24	520	
60	300	
130	000	
From	⊙ C	go E.
	To fence.	
50	1230	
70	1100	to ⊙ D
60	800	
20	480	
44	300	
90	000	
From	⊙ B	go N.
	To fence.	
66	2180	
90	2090	to ⊙ B
112	1600	
80	1000	
96	600	
60	000	
Begin	at ⊙ A	go W.

	to ⊙ C	
	2310	diag.
B 990	1830	
	720	1040 D
From	⊙ A	go N. W.
	To fence.	
96	1320	
106	1260	to ⊙ A
80	1000	
50	760	
100	400	
124	000	
From	⊙ D	go S.

Double areas.

4689300 Trap. A B C D

392220 } offsets on { A B
131920 } { B C
325016 } { C D
235280 } { D A

2)57·73736

28·86868 = 28*a.* 3*r.* 19*p.*
The whole area.

NOTE. The field in this case has also four sides, like the one proposed to be measured in the last example. Fields of this kind are often required to be measured in the rural districts, to ascertain the quantity of a growing crop, as grain, hay, turnips, &c., when sold by the acre; also to find the quantity of reaping, mowing, planting, &c.; in these cases the plan is seldom or never required, and the measurement is only taken as far as the growing crop extends, leaving out the hedges, ditches, and all other waste or other ground, not occupied by the crop in question.

6. Give the plan and area of a field from the following notes.

	to ⊙ D	
0	1134	
64	916	
134	728	
48	470	
86	246	
0	000	
	R. of ⊙ D	

	to ⊙ D	
0	2236	
258+0	930	
160+60	620	
130	400	
84	200	
0	000	
	R. of ⊙ C	

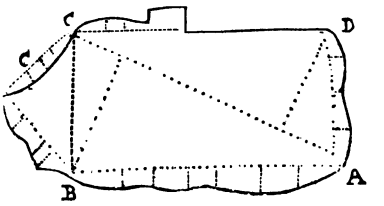
	to ⊙ C	
	1168	diag.
E 595	656	
	R. of ⊙ B	

	to ⊙ A	
0	2346	
74	2000	
88	1800	
156	1500	
92	1200	
170	800	
84	400	
0	000	
From	⊙ A	go W.

	to ⊙ B	
	884	0
	690	148
	512	50
	230	112
	000	0
	L. of ⊙ E	

	to ⊙ E	
0	774	
86	592	
118	400	
72	200	
0	000	
	L. of ⊙ C	

	to ⊙ C	
	2588	diag.
B 1060	2100	
	2000	
	1000	
	580	970 D
	R. of ⊙ A	

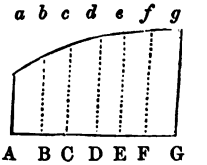


Double areas.

5253640	Trapezium A B C D	
684448	Triangle B E C	
445604	offsets on	A B
249580		C D
149304		D A
135400		E B
6917976	Insets on C E to be subtracted.	
107220		
2)68·10756		
34·05378 = 34*a*. 0*r*. 8*p*. Area.		

Problem VI.

TO FIND THE AREA OF A SEGMENT OF A CIRCLE, OR ANY OTHER CURVELINEAL FIGURE BY EQUIDISTANT OFFSETS OR ORDINATES.



Rule. If a right line A G be divided into any number of equal parts, AB, BC, CD, &c., and at the points of division perpendiculars be erected, A *a*, B *b*, C *c*, &c., to the curve *a b c d e f g*; then to the sum of the first and last offsets, add four times the sum of all the even offsets, and twice the sum of all the odd offsets, not including the first and last; multiply the sum by the common distance of the offsets, and one-third of the product will be the area, recollecting that the second, fourth, &c., are the even offsets, and the third, fifth, &c., are the odd offsets.

Note. If any portion of the figure is not included by an even number of offsets, its area must be found separately and added to the area found by the Rule.

EXAMPLES.

1. Required the plan and area of a piece of land measured by equidistant offsets or ordinates, from the following dimensions. (See last figure.)

	to ⊙ G	
g 450	600	G
f 442	500	F
e 430	400	E
d 406	300	D
c 376	200	C
b 306	100	B
a 212	000	A
From	⊙ A	go E.

First and last offsets.
1st. = 212 = A*a*.
Last = 450 = G*g*.

662 sum.

Even offsets.
2nd. = 306 = B*b*.
4th. = 406 = D*d*.
6th. = 442 = F*f*.

1154 sum.
4
4616 quadruple [sum.

Odd offsets.
3rd. = 376 = C*c*.
5th. = 430 = E*e*.

806 sum.
2
1612 twice sum.
4616
662

6890 sum total.
100 com. distance.

3)6·89000

Area. 2*a*. 1*r*. 7½*p*. = 2·29666

2. Required the plan and area of a piece of land, measured by equidistant offsets, the dimensions being as given below.

	to ⊙ B	
	1071	35 *c*
a	1000	48½ *d*
	900	69½
	800	87½
	700	103
	600	115
	500	124
	400	130
	300	132
	200	134
	100	131
	000	126½ *b*
From	⊙ A	go W.

175 First and last offsets.
2238 Four times even do.
933 Twice odd do.

3346 sum total.
100 common distance.

3)3·34600

1·11533 Area A *a d b*
2864 Trap. *a* B *c d*

1·14397 = 1*a*. 0*r*. 23*p*. Area.

Here the area of the trapezoid *a* B *c d* is found separately and added, agreeable to the preceding note.

3. Plan and find the area of a piece of ground from the equidistant offsets given below.

	to ⊙ B	
115	600	90
126	500	96
130	400	91
121	300	82
110	200	69
93	100	43
70	000	24
From	⊙ A	go E.

Here the piece of ground is curved on two sides, the base line A B passing nearly in the middle space between the two curves; in this case the sum of the offsets on each side of every distance must be considered as one offset, in finding the area.

4. Required the areas of two fields, the ends of which are straight and parallel, and the side curved by the following epuidistant offsets.

	to ⊙ B	
216	1200	200
150	1100	240
116	1000	256
100	900	255
77	800	260
65	700	259
67	600	250
80	500	229
100	400	202
120	300	180
160	200	144
202	100	95
277	000	50
From	⊙ A	go N.

Area 3*a*. 3*r*. 35*p*.

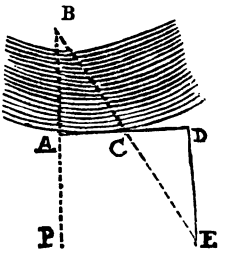
	to ⊙ B	
210	1097	209
180	1000	157
150	900	112
120	800	89
93	700	80
96	600	60
95	500	57
130	400	35
184	300	10
240	200	0
315	100	0
390	000	0
From	⊙ A	go W.

Area 2*a*. 2*r*. 13*p*.

PROBLEM VII.

TO MEASURE A LINE ACROSS A WIDE RIVER.

Let the annexed figure be a river, which is required to be crossed by the chain-line P B. Fix, or cause to be fixed, a pole or mark at B, at or near the margin of the river, in the line to be measured; erect the perpendicular A D, measuring A C and C D of any equal lengths; at D erect the perpendicular D E; on arriving at E, in the direction B C, the distance D E will be equal to A B, the required breadth of the river.



From the arrangement of the lines in the figure, it is evident that the triangles C A B, C D E are equiangular, and since A C was made = C D, the triangles are equal in all respects, and consequently A B = D E.

NOTE. A sufficient detail of methods of surveying by the help of the cross, which, though not much used by experienced surveyors, is a simple instrument, and its use readily understood by students. This method is, therefore, a proper introduction to the higher branches of surveying; besides, in rural districts, villages, &c., few surveyors use the more expensive instrument, the chain and cross being found quite sufficient to measure the quantities of growing crops, and other such small surveys as may be there required.

CHAPTER III.

LAND SURVEYING BY THE CHAIN ONLY.

This method of surveying has long been adopted by experienced surveyors; who have found it, in general, more accurate and expeditious, as well as better adapted to laying down extensive surveys, especially where no serious obstructions from woodlands, water, buildings, &c., exist; the use of the cross, in this method, being entirely excluded by some surveyors, and by others only used for secondary purposes, as for taking occasionally long offsets, or for squaring of lines obstructed by buildings, water, &c. Instead of the cross some use the Optical Square for these purposes; which will be hereafter described; while some erect perpendiculars with the chain only, as shall be shewn in the following preliminary Problems.

The fundamental lines of surveys of this kind usually form a large triangle, or several triangles, abutting from one common base, which ought, if possible, to extend throughout the whole length of the survey. The sides of the triangle, or triangles, must run as near as possible to the external and internal fences of the estate, or district, to be surveyed; the sides of each triangle being connected by one or more lines, running anywhere within the triangle, to determine the accuracy of the work. These lines are called proof or tie-lines; and where the estate to be surveyed contains a great number of inclosures, the proof-lines may almost always be found available in determining the positions of some of these inclosures. Where a great number of lines run within the main triangle, they are called secondary lines, and are usually numbered for the sake of reference. Some surveyors number the stations, or extremities of the lines; but the former method is here recommended. In small surveys, for preliminary instructions, the numbering of the lines is unnecessary, the stations being referred to by the letters of the alphabet, as already done in Chap. II.

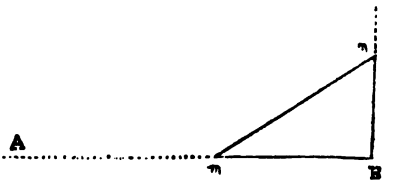
PRELIMINARY PROBLEMS.

Problem I.

TO ERECT A PERPENDICULAR WITH THE CHAIN.

Let *a* B be a chain-line, and A B the extended chain. It is required to erect a perpendicular to *a* B at B. Fix the end of the chain to the ground with an arrow at B; fix also the 80th link of the chain, reckoning from B, at *m*, 40 links from B; 80 links of the chain now lying slack between B and *m*. Take

hold of the 30th link of the chain from B, and extend it till it take the position B*nm*, the portions B *n*, *m n* of the chain being pulled tight; then shall B *n* be perpendicular* to the chain-line *a* B, and may be extended to any length required.

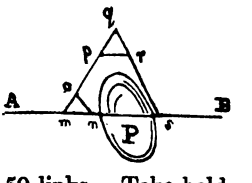


This method of erecting a perpendicular, though not so expeditious as that by the cross or optical square, is quite sufficient for those surveyors, who scarcely once require a perpendicular, in their operations, for weeks together; thus avoiding the inconvenience of daily carrying a cross, or other such like instrument for this purpose.

Problem II.

TO MEASURE A LINE IMPEDED BY AN OBJECT NOT OBSTRUCTING THE SIGHT.

Let A B be a chain-line, the direct measurement of which is prevented by the unforeseen obstruction of the pond P. Measure A *n* till it reach to, or near to, the edge of the pond, as to *n*, and fasten the ends of the chain to the ground with arrows at *m* and *n*, the distance *m n* being made half a chain or 50 links. Take hold of the middle of the chain, and extend it firmly, till its two halves rest in the positions *m o*, *o n*; thus making an equilateral triangle *m n o*, each side of which is 50 links. In the direction *m o*, measure to nearly opposite the middle of the pond, as to *q*. Again, make *p q* equal 50 links, fasten the ends of the chain at *p* and *q*, and extend its middle point to *r*, as before. In the direction *q r*, measure to *s*, till *q s* be equal to *m q*. Then *s* will be in the line A B, and *m s*† will be equal to *m q* or *q s*, which being added to A *m* will give the distance A *s*. Offsets being taken to the margin of the pond, during the measure-



* Since the parts B *n*, *n m* of the chain are together 80 links, of which B *n* is 30, the remainder *n m* is therefore 50; also B *m* was made 40; whence $40^2 + 30^2 = 50^2$, that is $m\,B^2 + B\,n^2 = m\,n^2$, therefore by Euc. I. 47, B *n* is perpendicular to B *m*, or *m* B *n* is a right angle.

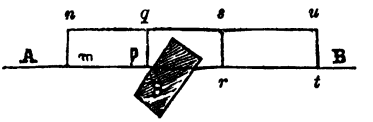
† Because the triangles *m n o*, *p q r*, are both equilateral, the angles at *m* and *q* are each 60° or one-third a right angle; whence by Theorem IV. the angle at *s* is also 60°; therefore all angles of the triangle, and consequently its sides, are equal, that is, $m\,q = q\,s = m\,s$.

ment of the lines $m\,q$, $q\,s$, and proper notes of the operation made in the field-book, the measurement from s to B may be continued.

Problem III.

TO MEASURE A LINE IMPEDED BY AN OBJECT OBSTRUCTING THE SIGHT, AS A BUILDING.

Let A B be a chain-line, the measurement of which is prevented by the building B. At m, four or five chains from the building, take a perdicular $m\,n$, of such a length that the line $n\,s$ may clear the building B. At or near the building take another perpendicular $p\,q$, exactly equal to $m\,n$, (these perpendiculars ought to be measured with the chain or a tape-line, if longer than the offset-staff,) and poles being put up, correctly vertical, at n and q, measure $q\,s\,u$ in the direction $n\,q$ of the poles, taking offsets to the building till it be cleared at s. Now on the line $q\,u$, at the distance $s\,u$, at or about equal to $m\,p$, erect the perpendiculars $s\,r$, $u\,t$, each exactly equal to $m\,n$ or $p\,q$, fixing poles, correctly vertical, at r and t. These poles are evidently in the true direction A B, and the measurement of the line may now be continued from r to B, after adding the distance $q\,s$ (which is equal to $p\,r$) to A p.



If the building, or other object, only protrude a few links over the line, the perpendiculars $m\,n$, $p\,q$, $s\,r$, &c., may be erected by the offset-staff, as nearly correct as can be judged by the eye, and the results will be sufficiently accurate.

Note 1. When an object, as a pond or pit, not obstructing the sight, protrudes only a short distance over the line (see last figure); it will be sufficient to erect only the two equal perpendiculars $p\,q$, $r\,s$ near its margin, with the offset-staff, as correctly as can be judged with the eye, and the distance $q\,s$, being measured, and added to A p, will give the distance A r.

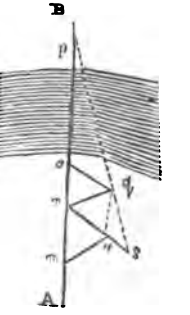
Note 2. Some unskilful surveyors square off the line, as they call it, when it is obstructed by a building, or other object, that impedes the sight, in the following manner. On arriving at or near the obstruction, as at p, a perpendicular $p\,q$ is erected to A p, another $q\,s$ is erected to $p\,q$; a third $s\,r$, equal to $p\,q$, is erected to $q\,s$; and lastly a fourth perpendicular r B to $r\,s$. Though this last perpendicular is theoretically in the line A B, the student will at once perceive that when so many as four perpendiculars are taken, one upon another, at a very short distance from one another, that slight inaccuracies in the observations, as well as in the perpendicularity of the poles, placed at p, q, s, will have an almost unavoidable tendency to derange the accuracy of the work; since a small error, made at the beginning, multiplies as the operation proceeds. But by the method, given in this Problem, the two perpendiculars, on each side of the obstruction, are placed so far apart, that a slight deviation in the perpendicularity

of the poles cannot materially affect the accuracy of the work, while a slight error in erecting the perpendiculars, provided their lengths be made exactly equal, will not affect the work in the slightest degree. If, therefore, ordinary care be take, the chance of error is almost impossible.

Problem IV.

TO FIND THE WIDTH OF A RIVER, WHICH IS TOO WIDE TO BE REACHED ACROSS BY THE CHAIN.

Let A B be the chain-line crossing a river, situated between o and p, a mark being fixed at p, on A B lay off $o\,n$, $n\,m$, each equal 50 links, and with the ends of the chain successively fixed at o, n, and at n, m, lay down the equilateral triangles $o\,q\,n$, $n\,r\,m$, as in Problem II., poles being fixed at n, q and r. In the two directions $p\,q$, $n\,r$, fix a pole at s, and measure the distance $r\,s$ accurately with a tape line to one-eighth of a link. Then by the similar triangles $s\,r\,q$, $q\,o\,p$, we shall have



$$r\,s : q\,r :: o\,q : o\,p.$$

But $q\,r = o\,q = o\,n = 50$ links, therefore,

$$r\,s : o\,n :: o\,n : o\,p = \frac{o\,n^2}{r\,s} = \frac{50^2}{r\,s} = \frac{2500}{r\,s}.$$

Whence the distance $o\,p$ becomes known.

For those who do not understand a rule, when symbolically expressed, we give, in words at length, the following.

Rule. Divide 2500 by the distance $r\,s$, and the quotient will be the breadth of the river, or the distance $o\,p$, which must be added to A o to give the distance A p.

EXAMPLES.

1. Required the breadth of a river by this method, when $r\,s$ measures 15 links.

$$\text{Here } \frac{2500}{15} = 166\tfrac{2}{3} \text{ links} = o\,p.$$

2. When $r\,s$ measures $13\frac{1}{8}$ links, required the breadth of the river.

$$\text{Here } 2500 \div 13\tfrac{1}{8} = \frac{2500 \times 8}{105} = \frac{4000}{21} = 190\tfrac{1}{2} \text{ links nearly.}$$

Problem V.

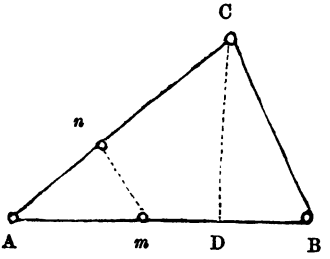
TRIANGULAR FIELDS.

When a triangular field, or piece of ground in that shape, is to be surveyed, set up poles or marks at each corner, and measure each side, leaving marks in at least two of the lines, and

entering their distances in the field-book; then measure the distance between the two marks for a proof line:—or, one mark only may be left in one of the lines, which may be connected with its opposite angle for a proof line.

EXAMPLES.

1. Required the construction and area of a field from the following dimensions.



Proof From	to ⊙ *n* 384 ⊙ *m*	line.
⊙ *n*	to ⊙ A 1244 700 L ⊙ C	
	to ⊙ C 852 L ⊙ B	
⊙ *m* From	to ⊙ B 1338 1000 600 ⊙ A	range E.

When the triangle A B C is constructed, the proof line *m n* will be found to measure 384 links, shewing that there has been no error in the work: also the perpendicular C D will be found to be 770 links; whence the area of the triangle $= 1338 \times 770 \div 2 = 5{\cdot}15130$ $= 5a.\ 0r.\ 24p.$ the area.

NOTE. If the proof line measured from the plan, does not exactly, or very nearly, agree with that measured in the field, some error has been made, and the work must be repeated.

TO FIND THE AREA OF A TRIANGLE FROM THE THREE SIDES.

RULE. From half the sum of the three sides subtract each side severally and reserve the three remainders; multiply the half sum continually by the three remainders, and the square root of the product will be the area.

NOTE. By this rule the area of a triangle may be found without laying it down, or finding the perpendicular.

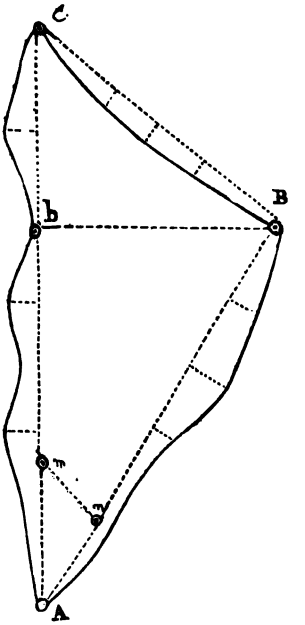
Adopting the preceding example, we have by the rule, $\frac{1338 + 852 + 1244}{2} = 1717 =$ half sum of the three sides. Then $1717 - 1338 = 379 =$ 1st remainder; $1717 - 852 =$ $865 =$ 2nd remainder; $1717 - 1244 = 473 =$ 3rd remainder; whence $\sqrt{(1717 \times 379 \times 865 \times 473)} = 5{\cdot}15022 = 5a.\ 0r.\ 24p.$

the sum as the area already found by measuring the perpendicular from the plan.

NOTE. This method of finding the areas of triangles is very little used in practice, on account of its requiring a tedious calculation, which may, however, be more readily performed by logarithms, as shall hereafter be shewn.

2. It is required to lay down a survey and find its content from the following field notes.

	to ⊙ A	
	2504	0
	2000	74
⊙ *m*	1860	351 to ⊙ *n*
	1650	137
	1430	90
	1220.	144
	850	30
	425	110
	000	0
	L. ⊙ C	
	to ⊙ C	
0	1346	
80	1072	
128	708	
98	458	
0	000	
	L. ⊙ B	
	to ⊙ B	
	1946	0
	1490	96
	1200	152
	1000	112
⊙ *n*	600	
	520	50
	000	
From	⊙ A	go N. E.



Having drawn the figure, the proof line *m n* will be found to measure 351 links, as in the field notes; and the perpendicular B *b* to be 1056 links.

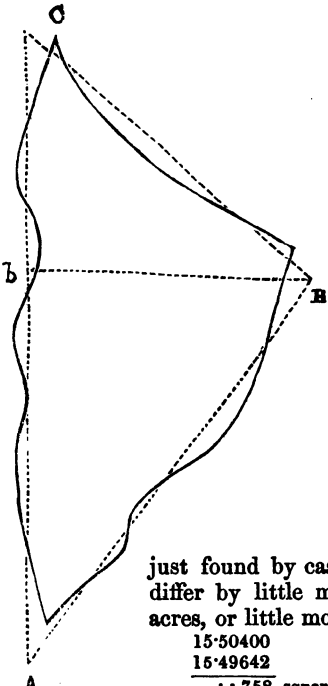
Double areas.

2644224	Triangle A B C
655676	Offsets on A B and A C
3299900	Sum
200616	Insets on B C
2)3099284	Difference

15·49642 = 15*a*. 2*r*. 0*p*. nearly, the area required.

COMPUTATION OF THE AREA BY CASTING, THAT IS, BY REDUCING THE CROOKED SIDES TO STRAIGHT ONES.

The offsets in the last example have been computed, in manner already shewn in Chap. I.; but by this method straight lines are drawn on the plotted figure so as to include as much space in the area to be measured, as they exclude, as nearly as can be judged by the eye, the area to be measured is thus reduced to a figure bounded by right lines only, which may thence be much more expeditiously reduced to triangles, trapeziums, &c. The method of drawing these lines is usually by a straight edged ruler of transparent horn, or by a silken thread stretched with a bow; the ruler or thread being moved over the crooked fence, till it appear to the eye to enclose as much of the adjoining ground as is left out, a line is then drawn in this position; and so on for other crooked fences. Thus the trouble of calculating numerous offsets is completely avoided, and with proper care equal accuracy is obtained.



2. We shall adopt the last example for this method of casting, that it may be seen how near the two methods agree.

The figure being constructed, and the boundary drawn carefully with ink, the chain-lines must then be rubbed out, and the three dotted lines A B, B C, C A must now be drawn, in such a manner, that the parts excluded by them may be equal to the parts included, as nearly as can be judged by the eye. The base A C will be found to be 2584 links, and the perpendicular B b = 1200 links. Whence

$$\frac{2584 \times 1200}{2} = 15{\cdot}50400 = 15a.\ 2r.\ 1p.$$

nearly, *the area by this method.*

If the area found by the true method be taken from the area, just found by casting, it will be seen that they differ by little more than one pole out of 25½ acres, or little more than 1 in 4000: thus

15·50400
15·49642
··758 square links, or little more than one pole.

NOTE. It will hence be seen at once that a great deal of trouble is saved by this method, which is therefore generally adopted by practical surveyors; although it is certainly less correct than by calculating from the offsets, the former method depending chiefly on the accuracy of the casting lines for the truth of its results; but practice will soon render it easy to draw the lines so as to obtain almost perfect accuracy.

3. Required the plans and contents of two fields by both the methods of calculation, viz. offsets and by castings, from the following field-notes.

	To ⊙ B	
	1151	Proof line.
From	⊙ D	
	to ⊙ A	
	2640	
	1200	⊙ D
	R. ⊙ C	
	to ⊙ C	
0	1760	
63	1600	
145	1500	
190	1400	
117	1200	
120	1100	
189	1000	
120	760	
127	500	
120	400	
90	200	
111	000	
	R. ⊙ B	
	To fence.	
61	1861	
34	1750	⊙ B
95	1600	
129	1400	
150	1200	
130	1000	
115	800	
110	600	
60	400	
33	200	
0	000	
From	⊙ A	go E

Content 18*a*. 3*r*. 33*p*.

	To ⊙ A	
0	2160	
13	2000	
99	1800	
220	1600	
259	1400	⊙ D to ⊙ B 1193 proof line.
291	1245	
212	1100	
159	1000	
61	900	
32	820	
71	750	
109	600	
140	500	
147	400	
173	300	
140	200	
93	100	
0	000	
	R. ⊙ C	
	to ⊙ C	
0	1696	
61	1600	
119	1500	
161	1400	
140	1300	
96	1200	
40	1040	
91	900	
179	760	
0	660	
19	540	
140	400	
80	300	
98	200	
92	000	
	R. ⊙ B	
	To fence.	
	1598	
	1510	⊙ B
From	⊙ A	go E.

Content 17*a*. 2*r*. 27½*p*.

NOTE. It will be seen that in the main triangles of these two surveys, the proof lines have been taken from a side of each to its opposite angle; which is the best method of proof, when convenient to make it; but it may be performed with equal accuracy by taking a proof line from one side to another, at a short distance from one of the angles of the triangle.

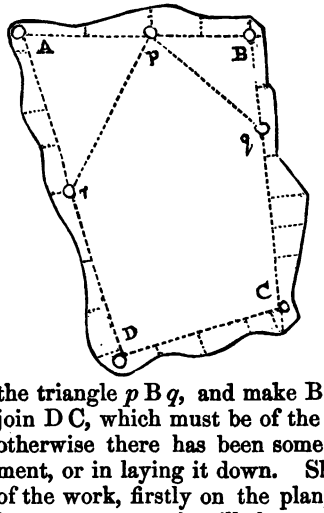
PROBLEM VI.

FOUR SIDED FIELDS.

When a field has four sides, straight or crooked, measure the four sides, or lines near them, if crooked, taking the offsets: also measure one or both the diagonals, one of which will serve as a base in plotting the work, and the other for a proof-line; or the proof-line may be measured in any other direction that may be most convenient.

Sometimes the measurement of both the diagonals is prevented by obstructions, in such cases it will be sufficient to measure tie-lines across two of the angles of the trapezium, at the distance of from two to five chains from each angle, according to the size of the field. These tie-lines with their distances from the angles on the main-lines will be found sufficient for planning the lines and proving them.

EXAMPLES.



1. In the annexed figure the lines A B, B C, C D, D A are measured, marks being left at *p*, *q*, and *r*, and their respective distances on the lines noted in the field-book, thus furnishing the follow method of laying down the plan.

On A B, as a base, take A *p* = given distance, and with the distances A *r*, *p r*, and centers A and *p* describe arcs cutting in *r*; then prolong A *r*, and lay off thereon the given length A D. In the same manner construct the triangle *p* B *q*, and make B C = its given length. Lastly, join D C, which must be of the length shewn in the field-book, otherwise there has been some mistake either in the measurement, or in laying it down. Should this be the case the whole of the work, firstly on the plan, and secondly in the field, must be gone over again till the error be discovered.

NOTE. When the main lines that include the chief part of the ground to be

measured are of considerable length, as from 30 to 40 chains, it will be necessary to take the tie-lines at least 10 chains from the angles, across which they are measured; for a small error, in laying down the plan with short tie-lines, will cause the main lines to deviate considerably from their true position when prolonged. However, it sometimes happens that long tie-lines cannot be obtained in consequence of obstructions. In such cases the tie-line must be carefully measured to even one-fourth of a link; the distance of each tie-line from its angle and the tie-line itself must then be all multiplied by 4, thus throwing fractions out of the question, and with the three lines, thus increased, the triangle determining the position of an angle of the trapezium, may be accurately constructed. The proof-line and its distances from its angle must be similarly treated, that the accuracy of the work may be fully established.

2. Required the plan and area of a straight-sided field from the following dimensions.

To ⊙ *n* 481 Proof-line.....	To ⊙ A 952 452 L. ⊙ D	⊙ *m*
To ⊙ B 833 Proof-line.....	to ⊙ D 1236 400 L. ⊙ C	⊙ *r*
	to ⊙ C 886 L. ⊙ B	
⊙ *n* From	to ⊙ B 1446 500 ⊙ A	go North.

When the figure has been laid down, the diagonal A C will be found = 1962, and the perpendiculars thereon from B and D respectively = 632 and 514 links. Whence the area is 11*a.* 0*r.* $5\frac{3}{11}p$.

3. Draw the plans, and find the contents of two enclosures, from the following field-notes, both by calculation from the offsets and by casting.

NOTE. In each of the two following examples, it will be seen that there are two straight sides, and two that require offsets: also, in the former example, one of the crooked sides is crossed by the chain-line, thus producing insets, the content corresponding to which must be subtracted, as in former cases.

	To ⊙ B	
	2921	diag.
	2000	
	1000	
Return	to ⊙ B	
	to ⊙ C	
	2960	diag.
	2000	
	1000	
	R. ⊙ A	
	to ⊙ A	
	1344	
	1000	
	R. ⊙ D	
	to ⊙ D	
0	2488	
93	2000	
140	1600	
171	1200	
135	800	
60	400	
0	000	
	R. ⊙ C	
	to ⊙ C	
0	1440	
171	1300	
223	1100	
	1000	
175	900	
Cross	721	fence
	600	165
	400	261
	200	199
	000	0
	R. ⊙ B	
	to ⊙ B	
	2700	
	2000	
	1000	
From	⊙ A	go W.

Content 38*a.* 1*r.* 3½*p.*

	To ⊙ D	
	2473	diag.
	2000	
	1000	
Return	to ⊙ B	
	to ⊙ C	
	2652	diag.
	2000	
	1000	
	R. ⊙ A	
	to ⊙ A	
0	1030	
57	800	
99	600	
130	400	
106	200	
0	000	
	R. ⊙ D	
	to ⊙ D	
0	2750	
101	2600	
145	2400	
168	2000	
104	1600	
0	1310	
0	715	
88	400	
99	200	
0	000	
	R. ⊙ C	
	to ⊙ C	
	1328	
	1000	
	R. ⊙ B	
	to ⊙ B	
	1892	
	1000	
From	⊙ A	go E.

Content 28*a.* 0*r.* 19½*p.*

PROBLEM VII.

FIELDS HAVING MORE THAN FOUR SIDES.

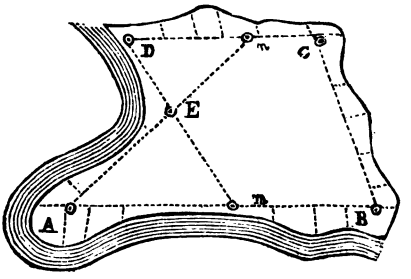
Various methods will suggest themselves to the surveyor for taking lines to lay down a field that requires more than four main lines to take its boundary. The method of dividing such

fields into trapeziums and triangles is, in most cases, circuitous, and displays little skill on the part of the surveyor, especially where all the sides are crooked, and where a plan is required. A few methods of surveying fields of this kind will, therefore, be presented to direct the student; although their variety of shape is so endless, that no general rule can be given for laying out lines on the ground, that shall give an incontestably accurate plan. To tie every angle in succession, though true in principle, is by no means a safe method, especially where there are a great number of angles to be tied, as an error in one of the tie-lines will derange the whole of the work, without affording the means of detecting where the error lies.

NOTE. The following examples of surveys of this kind occurred in part of the author's extensive practice, as a surveyor of parishes, under the Tithe-Commissions. The student is recommended to sketch the following specimens on a large scale, and find their contents by the usual methods.

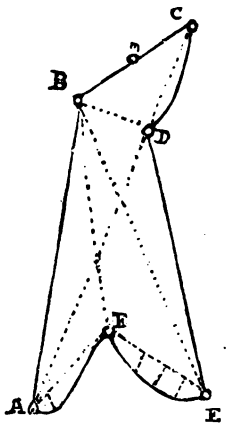
EXAMPLES.

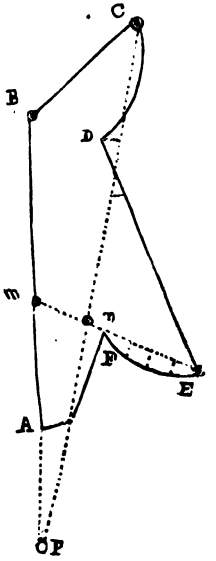
1. Here a field of five sides is surveyed by the same number of lines, viz. AB, BC, CD, D*m*, and A*n*, the last two intersecting in E. These lines evidently constitute a decisive proof among themselves, and all of them are available in taking the boundary.



In surveying this field (poles or natural marks being supposed to be fixed at A, B, C, D, and E) commence close to the river's edge, in the line A B prolonged backwards, enter the offsets and the station A in the field-book. On arriving at ⊙ *m*, in the direction ED, enter its distance, and so on to ⊙ B, measuring the line to the fence; from B proceed to C, in like manner, measuring beyond the station to the fence. The place of the ⊙ *n* is to be noted, on arriving in the direction EA, while measuring C D. D*m* is next measured, the place of the ⊙ E being noted. Lastly, go from *m* to E, and measure A*n*, entering the place of the ⊙ E a second time, all the offsets being supposed to be taken during the operation.

Construction of the plan. Select the distances A*m*, A E, and E*m* from the field-book, and with them construct the triangle A*m* E, prolong the sides to their entire lengths, up to the boundaries, and fix the places of the stations B, *n*, and D.





	To ⊙ B	
	1321	
Return	to ⊙ F	diag.
	to ⊙ E	
	1939	diag.
	L. ⊙ B	
	to ⊙ B	
	536	diag.
	L. ⊙ D	
	to ⊙ D	
	1664	diag.
	R. ⊙ A	
	to ⊙ A	
0	569	
69	400	
0	000	
	L. ⊙ F	
	to ⊙ F	
0	767	
81	600	
105	400	
65	200	
0	000	
	R. ⊙ E	
	to ⊙ D	1607 to ⊙ E, length of fence.
0	766	
6	400	
0	000	
	R. ⊙ C	
	to ⊙ C	
	950	
	600	⊙ *m* to ⊙ D 511 proof line.
	R. ⊙ B	
	to ⊙ B	
	1699	
	1000	
From	⊙ A	range North.

Area 13*a.* 1*r.* 7*p.*

It will be seen, from the field-notes and the first figure, no less than eleven lines are used to complete the survey by the old method; whereas four lines will be found amply sufficient to effect the same purpose: thus the three lines PB, BC, and CP, constituting the triangle PBC, are first measured, stations being left at *m* and *n* in the direction of the corner at E; the

line m E is then measured through the ⊙ n; thus proving the triangle P B C, and determing the position of the straight fence D E, which position is further proved by offsets and a crossing at, and south of D, the crooked fences having been taken by offsets in the usual way: thus completing the survey with little more than one third of the lines required by the former method.

5. Required the plans and contents of two straight-fenced fields, each having five sides, from the following field notes, as given by old methods.

	To ⊙ A	
	2083	diag.
	1000	
Return	to ⊙ D	
	to ⊙ E	
	840	diag.
	R. ⊙ C	
From	to ⊙ C	to ⊙ m 519.
	1537	diag.
	1000	
	R. ⊙ A	
	to ⊙ A	
	1170	
	R. ⊙ E	
	to ⊙ E	
	1035	
	R. ⊙ D	
	to ⊙ D	
	730	
	L. ⊙ C	
	to ⊙ C	
	779	
	R. ⊙ B	
	to ⊙ B	
	2107	
	2000	
	1200	⊙ m
Begin	at ⊙ A	go W.

Content 12a. 2r. 5p.

	To ⊙ B	
	755	diag.
	R. ⊙ E	
	to ⊙ E	
	619	diag.
From ⊙ m	200	to ⊙ D 142
	R. ⊙ C	
	to ⊙ C	
	599	diag.
	R. ⊙ A	
	to ⊙ A	
	196	
	R. ⊙ E	
	to ⊙ E	
	346	
	R. ⊙ D	
	to ⊙ D	
	309	
	R. ⊙ C	
	to ⊙ C	
	267	
	R. ⊙ B	
	to ⊙ B	
	667	
Begin	at ⊙ A	range W.

Content 1a. 2r. 36p.

NOTE. In the two preceding examples, nine lines are required in each case, by the circuitous methods there adopted. Each field, by the improved methods, here laid down, may be surveyed by five lines, or both fields by only one line

more than are required by the methods given in the preceding field notes. Precisely the same system of lines may be adopted in the former case, as has been done in the Example 1. The latter case is left to the ingenuity of the student.

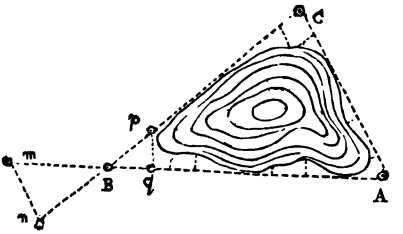
PROBLEM VIII.

WOODS, LAKES, AND SWAMPY GROUNDS.

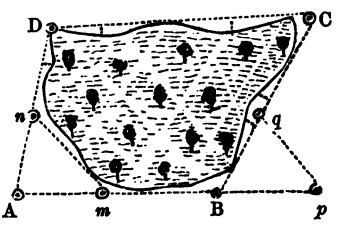
When woods, lakes, and swampy grounds are required to be surveyed, where lines cannot be measured upon them, a system of lines must be adopted for each particular case, so combined by triangulation as to prove their accuracy, among themselves, when laid down on paper. If the wood, or other inaccessible space, (as far as measuring is concerned) be either of, or very near, a triangular shape, the three sides of a triangle will compass it, which may be proved by a tie-line at one of its corners, if the wood or lake will admit one of sufficient length, but, if not, any two of the sides of the triangle may be prolonged for this purpose, offsets, or rather insets, being taken to the boundary of the wood or lake, in the usual way.

EXAMPLES.

1. Here the three sides of the triangle A B C compass a lake or large pond, insets being taken therefrom to the margin of the water. The accuracy of the work is proved by the tie-line *pq*, or, if this line be thought too short, (as it apparently is) the sides AB, CB may be prolonged to *m* and *n*, till B *m*, B *n*, each equal about one third of AB, and the tie-line *m n*, being measured, will establish the accuracy of the work, or prove it wrong, as the case may be.



2. The annexed figure shews the survey of a wood, which is effected by the four main-lines AB, BC, CD, DA, the first line being prolonged to *p*, and stations being left at *m*, *n*, and *q* for the tie-lines *mn*, *pq*. It scarcely need be added that when the lines AB, AD, BC, are laid down by the help of the two tie-lines, the line BD will exactly fit in, if all the work has been accurately done.



3. Required the plan and area of a wood from the following field notes, two of the fences of which are straight.

	To ⊙ *p*	
	1175	
⊙ C 0	825	
28	600	
0	500	
	400	⊙ *n* 491
		to ⊙ *m*
0	275	
20	150	
52	25	
0	000	
Return	to ⊙ B	

	to ⊙ *m*	
	900	
⊙ B 0	600	
25	500	
0	425	
0	300	
50	200	
40	125	
0	000	
From	⊙ A	go E.

	To ⊙ A	
0	812	
	⊙ D	
	to ⊙ D	
	575	
	400	⊙ *q* 550
Return	to ⊙ C	to ⊙ *p*

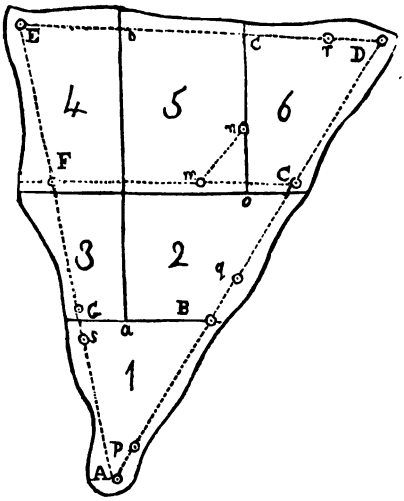
The figure being laid down, and reduced by casting the two crooked sides, the area will be found 458065 square links, or 4*a*. 2*r*. 13*p*.

NOTE. As a general rule for surveying woods and lakes, the following may be given:—Measure as many lines round the boundary to be surveyed as will compass it, and tie all the angles, except the two last, as in the preceding examples; you will thus obtain a system of lines that will prove among themselves, as the last measured side will just reach from the last station to the first, if the work have been done with accuracy. But the shapes of woods, &c., are so various that it would not be adviseable, in every case, to adhere to this method: much must, therefore, be left to the skill of the surveyor. In the southern counties of England, where coppice wood is sold by the acre for fuel, it is very frequently required to survey a portion of a wood, (the coppice being cut down, and the large timber still standing); in such cases, the lines must be taken within the space to be surveyed, as the adjoining uncut coppice prevents their being taken outside. The lines must, therefore, be ranged among the growing timber, as well as they can; and the tie lines taken through the most convenient openings left by the trees. Surveys of this kind, it may be proper to add, are best performed with the help of the Prismatic Compass, or the Box Sextant, which are sufficiently accurate for these surveys, which are always of small extent.

Problem IX.

TO SURVEY A SMALL ESTATE, DIVIDED INTERNALLY BY STRAIGHT FENCES.

A small estate, of a form nearly triangular, is divided into six fields by four straight fences, as shewn on the annexed plan. The survey is commenced at A, by measuring the line A D, taking the offsets, and leaving stations at the crossing of these fences at B and C. The line D E is now measured, also crossing two of the straight fences. From the ⊙ E the line E A is measured, leaving stations at the hedge-crossings F and G: thus completing the triangle A D E. Next, the line B G is measured, close to the straight fence B G, and crossing the straight fence *a b*. Lastly, the line F C is measured, again crossing the straight fences *a b* and *c o*: a station being left at *m*, about a chain's distance from *o*, and one chain measured close by the fence *o c* to *n*, and from thence to ⊙ *m*: (the last operation being, in most cases, the most readily performed before the line passes the fence *c o*) thus completing the survey. The position of the straight fence *a b* is proved to be correct by the crossing of three of the lines of the survey, viz., B G, F C, and D E; the crossings made by these three lines on *a b*, must be in a right line on the plan, otherwise there has been an error in the field notes. The straight fence *o c* has only two crossings by the surveying lines F C, D E, its position is, therefore, not duly proved to be correct without the tie line *m n*, measured in the manner already stated. This expeditious method of proving the position of a straight fence, crossed by two chain lines (as the fence *c o*), was never adopted till done so by the author, in the parish survey under the *Tithe Commissioners, who, in their instructions to surveyors, directed the lengths of all such fences to be measured to establish the accu-*



racy of their positions. Now the measurement of the length of such a straight fence as *c o*, which is nearly perpendicular to the two chain lines that cross it, gives a very imperfect proof of the accuracy of its position, especially if it be a long one. In this latter case the fence *c o* might be a full chain from its true position at one end, and its length, as shown by the plan, would be so near the measured length that the error would not be detected; whereas the tie line *m n*, shown on the plan, would detect the error at once, by its increased or diminished length. However, when two chain lines cross a straight fence obliquely, the length of the fence, from crossing to crossing, evidently gives a sufficient proof of the accuracy of its position; yet to measure a short tie line, at one of the crossings, is a shorter method of proof, in all cases where the length of the fence is considerable. The method of proof recommended by the Commissioners in question, ought, therefore, never to be adopted, except where one or both of the chain lines cross the straight fence obliquely, and at a short distance. It seems hardly necessary to remark that the positions of the straight fences G B, C F are determined by the lines measured close to them. *Thus the survey of six fields may be made, and its accuracy proved, by five lines, with the two short tie lines m n, n o, which may be regarded as mere offsets. Besides, had the fences* G B, C F *been crooked, the same lines would have effected the survey by offsets thereon. Moreover, the survey of these six fields, all the internal fences being as shown in the plan, may be accurately effected by four lines, in the following manner.* The triangle A D E remaining as the foundation of the survey, let a station, or rather direction point, *p* be entered in the field notes, in the direction of the straight fence *a b*, and another similar point at *q*, in the direction of the straight fence *o c;* leave also a proper station mark, or pole, at *r;* on arriving at *s* in E A, leave another station, in such a position, that a line from *s* to *r* will cross all the four straight fences. This last line will prove the fundamental triangle A D E and the positions of the four straight fences, at the same time, without measuring the lines by the fences G B, C F; which can have now three crossings by chain lines, and the other two fences *a b*, *o c* have each two crossings by chain lines, and each one direction point, viz: *p* and *q*; through which points these fences must respectively pass, after they have been drawn through their crossings on D E, and the other line from *s* to *r*, not shewn on the plan. This method of determining the position of straight fences, though theoretically elegant, cannot always be easily

practised, especially where the fences are high, or the ground hilly, thus preventing the directions of the straight fences being seen to distant chain lines, as in the cases of the fences *a b*, *o c*, with respect to the chain line A D.

THE METHOD OF MEASURING HILLY GROUND.

When the ground, over which lines are measured, rises or falls, or both alternately, the horizontal distances are what are required in plotting the survey, as well as for finding the content thereof, and not the actual distances measured along the surface of the ground.

For many ordinary purposes the horizontal measurement may be obtained by holding the end of the chain up so as to keep it horizontal, as nearly as can be judged by the eye, the arrow being placed vertical under the end so held up: but when a large and accurate survey is required, the distances must be measured along the line of ground, and the angles of elevation and of depression of the several inclined parts of the line must be taken, either with a common quadrant, or afterwards with the theodolite (to be hereafter described), and the lengths of the several rises and falls must be noted; from which the corresponding correct horizontal distances may be readily computed. The following table shews the number of links to be subtracted from every chain, or 100 links, for the angles there set down.

TABLE shewing the reduction in links and decimals of a link upon 100 links for every half degree of inclination from 3° to 20° 30′.

Angle.	Reduction.	Angle.	Reduction.	Angle.	Reduction.
3° 0′	0·15	9° 0′	1·23	15° 0′	3·41
30	0·19	30	1·37	30	3·64
4 0	0·24	10 0	1·53	16 0	3·87
30	0·31	30	1·67	30	4·12
5 0	0·38	11 0	1·84	17 0	4·37
30	0·46	30	2·01	30	4·63
6 0	0·55	12 0	2·19	18 0	4·89
30	0·64	30	2·37	30	5·17
7 0	0·75	13 0	2·56	19 0	5·45
30	0·86	30	2·76	30	5·74
8 0	0·97	14 0	2·97	20 0	6·03
30	1·10	30	3·19	30	6·33

By this table the trouble of computation is avoided, only the

distance, measured on each rise or fall, requiring to be multiplied by the reduction in chains corresponding to the angle of each rise or fall, and the product, subtracted from that distance, will give the correct distance, as in the following

EXAMPLES.

1. A line was measured 12·43 chains, on ground having a rise of 8½ degrees, required the horizontal length of the line.

Here to 8½°, or 8° 30′ corresponds the reduction 1·10 links, whence

12·43		Whencc 12·43	
1·10		13·673	
13·6730		12·29·327	horizontal distance,

in which the decimal, being less than half a link, is rejected; thus making the correct horizontal distance 12·29 chains, or 1229 links.

2. The acclivity of a hill rises 20°, and measures 16·14 chains, its declivity falls 11½° and measures 32·28 chains, required the horizontal distance between the extremities of the line thus measured over the hill, it being level at the top 2·80 chains,

Here $16{\cdot}14 \times 6{\cdot}08 = 98{\cdot}1312$
$32{\cdot}28 \times 2{\cdot}01 = 64{\cdot}8828$

163·0140 or 163 links, or 1·63 chains.

Whence 16·14
32·28
2·80

51·22
1·63

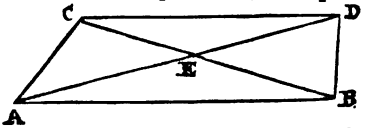
49·59 chains, the horizontal distance required.

NOTE. When fences are crossed, stations made, &c., on the acclivity or declivity of a hill, the horizontal distance up to such points must be found. Some surveyors place the arrow forward a distance equal to reduction, due to the angle of acclivity or declivity, at the end of every chain measured, and thus obviate the necessity of reducing the line afterwards, having for the purpose a small pocket quadrant, so graduated that the plumb-line thereof shews, on observing the angle of elevation or depression, the reduction required for each chain. A great deal of trouble is thus saved, as the theodolite cannot be conveniently carried about for this purpose. Such pocket quadrants are not made by the mathematical instrument makers, being the productions of clock makers or other mechanics, according to the various designs of surveyors.

THE USE OF THE PARALLEL RULER,

IN REDUCING CROOKED FENCES TO STRAIGHT ONES, TO FACILITATE THE COMPUTATION OF THE CONTENTS OF FIELDS.

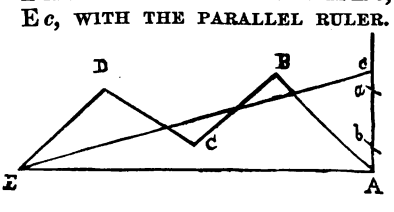
As some surveyors prefer the parallel ruler to the method already given, for reducing crooked fences to straight ones, the method of using that instrument for this purpose is here given. This method is founded on a well-known proposition of Euclid, in which it is shewn that *triangles on the same base, and between the same parallels, are equal.*



Let A B C, A B D be triangles on the same base A B, and between the same parallels A B, C D; then the triangle A B C is equal to the triangle A B D. And, if the triangle A E B, which is common to the other two triangles, be taken away, the remaining triangles A E C, B E D will also be equal; whence equal areas may be transferred from one side of a line to the other, which is the principle, on which, as already said, the following Problems are founded.

PROBLEM I.

IT IS REQUIRED TO REDUCE THE OFFSET-PIECE A B C D E TO A RIGHT ANGLED TRIANGLE A E *c*, BY AN EQUALIZING LINE E *c*, WITH THE PARALLEL RULER.



Draw the indefinite line A *c* perpendicular to A E. Lay the parallel ruler from A to C; hold the near side of the ruler firmly, and move the further side to B, which will cut A *c* at *a*, where a mark must be made. Lay the ruler from *a* to D, and the further side thereof being now held fast, bring the near side to C, marking A *c* at *b*. Lay the ruler from *b* to E, move it parallel to D, marking A *c* at *c*. Join E *c*; then A E *c* is a right angled triangle required, and its area may be found by taking half the product of A E and A *c*.

THE FOLLOWING IS A GENERAL RULE FOR SOLVING PROBLEMS OF THIS KIND.

Draw a temporary line, as A *c* at right angles, or at any other angle to the chain line, as A E, of the offsets.

1. Lay the ruler from the first to the third angle, and move it parallel to the second angle; then make the first mark on the temporary line.

2. Lay the ruler from the first mark on the temporary line to the fourth angle, and move it parallel to the third angle; then make the second mark on the temporary line.

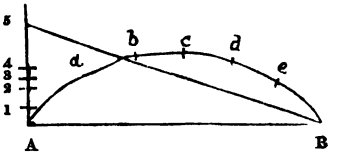
3. Lay the ruler from the last named mark to the fifth angle, and move it parallel to the fourth angle; then make the third mark on the temporary line.

4. Lay the ruler from the last named third mark on the temporary line to the sixth angle, and move it parallel to the fifth angle; then make the fourth mark on the temporary line.

In this manner the work of casting by the parallel ruler may be conducted to any number of angles. Great care must be taken, during the operation, to prevent the ruler slipping, as such an accident will derange the whole of the work, if not discovered and immediately corrected.

Problem II.

TO REDUCE A CURVED OFFSET-PIECE TO A RIGHT-ANGLED TRIANGLE.



Let A *a b c d e* B be the curved offset-piece. Divide the curve by points *a*, *b*, &c., so that the parts A *a*, *a b*, &c., may be straight, or nearly so; and draw A 5 perpendicular to A B. Lay the ruler from A to *b*; move it parallel to *a*, and mark A 5 at 1. Lay the ruler from 1 to *c*; move it parallel to *b*, and mark A 5 at 2. Lay the ruler from 2 to *d*; move it parallel to *c*, and mark A 5 at 3. Lay the ruler from 3 to *e*; move it parallel to *d*, and mark A 5 at 4. Lay the ruler from 4 to B; move it parallel to *e*, and mark A 5 at 5. Draw the line B 5; then will A B 5 be a right angled triangle equal in area to the offset-piece A *a d c d e* B, as required.

EXAMPLES FOR PRACTICE ON THE TWO PRECEDING PROBLEMS.

1. Lay down a right-lined offset-piece, from the following notes; reduce it to a triangle by the parallel ruler; and find its content, both by calculation from the offsets and the casting of the ruler.

	To ⊙ B	
	751	0
	550	150
	400	51
	250	99
	50	75
	000	0
From	⊙ A	range West.

The area found by calculation from the offsets is 0*a*. 2*r*. 17*p*. and the perpendicular of the triangle, found by the casting of the parallel ruler, is 161 links. Hence $\frac{751 \times 161}{2} = 0{\cdot}60455$ square links, or somewhat more than 0*a*. 2*r*. 16½*p*. It hence appears that the method of casting by the ruler, gives the content very near the truth. In fact this method is mathematically accurate; but the danger of error, as already said, arises from the accidental slipping of the ruler during the operation.

2. Lay down and find the area of a curve-lined offset-piece, from the following field notes; and find its area both by calculation, and by the casting of the parallel ruler.

	To ⊙ B	
0	600	
95	500	
131	400	
134	300	
126	200	
89	100	
0	000	
From	⊙ A	range East.

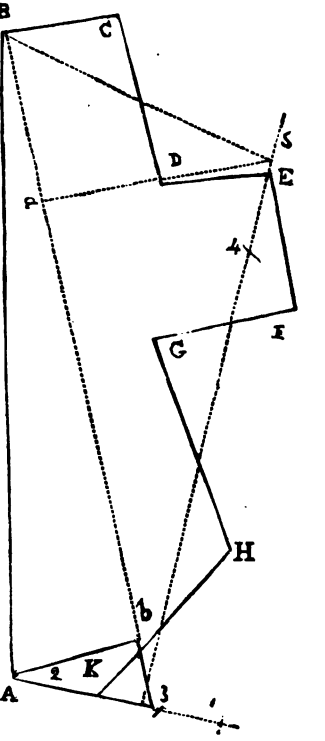
By both the methods the area is found to be 0*a*. 2*r*. 15½*p*. nearly, the perpendicular on A B being found 199 links by the casting operation.

Problem III.

TO REDUCE THE IRREGULAR FIELD A B C D E F G H K TO A TRAPEZIUM BY THE PARALLEL RULER.

Prolong the line A K at pleasure. Lay the ruler from K to G; move it parellel to, and mark A K prolonged at 1. Lay the ruler from 1 to F; move it parallel to G, and mark A K

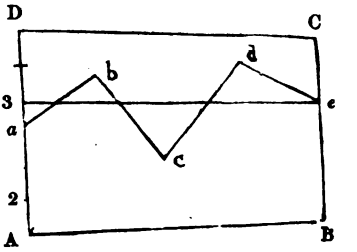
at 2. Lay the ruler from 2 to E; move it parallel to F, and mark A 1 at 3. Draw a line 3 to E and prolong from E. Lay the ruler from E to C; move it parallel to D, and mark 1 E at 4. Lay the ruler from 4 to B; move it parallel to C, and mark 1 E prolonged at 5. Draw a line from 5 to B; then shall A B 5 3 be a trapezium, equal in area to the irregular figure A B C D E F G K; the area of which may be found by multiplying the diagonal B 3 by half the sum of the perpendiculars thereon from A and 5.



NOTE. In this manner the crooked sides of a field may be successively reduced to straight ones. Thus, if the side A B had been crooked, the operation of straightening might be continued by prolonging the dotted line 5 B, and find successive points therein, corresponding to the assumed angles, till the last angle was brought thereon, and so on with respect to the side A K, had it also been crooked. When the sides of a field are curved, the method of reducing them to straight lines is the same as shewn in Problem II.

PROBLEM IV.

TO DRAW AN EQUALIZING LINE THROUGH THE CROOKED FENCE *a b c d e*, SO THAT THE TWO FIELDS A B *e a*, *a* D C *e* MAY BE FOUR SIDED.



Lay the ruler from *a* to C; move it parallel to *b*, and mark A D at 1. Lay the ruler from 1 to *d*; move it parallel to *c*, and mark A K at 2. Lay the ruler from 2 to *e*; move it parallel to *d*, and mark A K at 3. Draw the line *e* 3, and it will divide the two fields, so that their quantities shall be the same as those before separated by the crooked fence *a b c d e*.

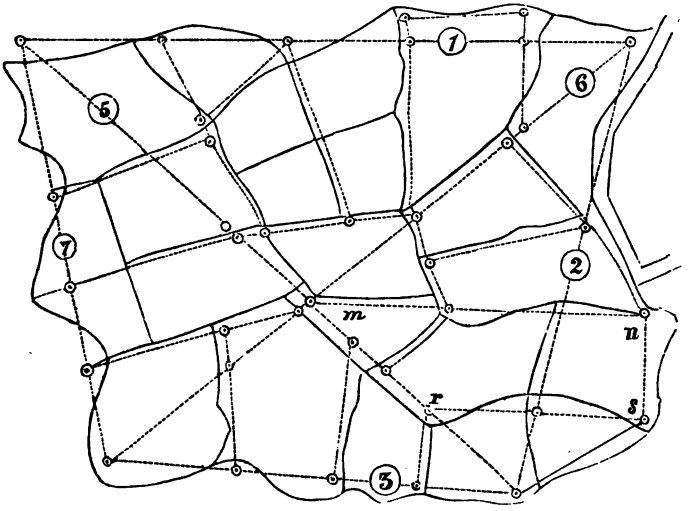
It is scarcely necessary to add, that had the fence *a b c d e* been curved, the equalizing line might have been found as in Prob. II.

NOTE. The owners of adjoining estates sometimes agree to straighten fences or boundaries, by an equalizing line of fence, or, as they term it, "by giving and taking equal quantities of land." When this is required to be done, the crooked or curved boundary fence must be first measured on the ground, and then plotted on a large scale; when the equalizing line may be drawn on the plan, as shewn in the last Problem, and the distance from D to 3, or from *a* to 3, must be correctly measured by the scale: this distance must next be measured in the field, accordingly as it is taken from D or *a*; and the new line of boundary *e* 3 may now be ranged and marked out, preparatory to making the required fence. Moreover, to guard against errors, in the preceding operation, it is advisable to measure, both on the plan and on the ground, the parts cut off, on each side of the new boundary line, thus proving the work; as an error is of serious consequence in these matters: and, if one is found to exist, it must be corrected before the boundary fence is made.

SURVEYING LARGE ESTATES OR PARISHES BY THE CHAIN ONLY.

1. Having perambulated the boundary of the estate, parish, or lordship to be surveyed, if you find that its boundary approaches somewhat near to that of a four-sided figure, or trapezium, the system of fundamental lines, adopted by order of the Tithe Commissioners of England and Wales, is to be preferred. These fundamental lines are six in number, of which four must run close by, or as nearly as possible to, the boundary in question, thus forming a trapezium, four lofty station poles being placed at each angle, as objects for running the lines; the other two lines must form the diagonals of this trapezium, and therefore pass through the central parts of the survey, intersecting each other, the points of intersection being noted on measuring each line, so that when the system of lines are laid down on the plan, the proof of the accuracy of the work may be fully established, before the minor operations, or filling up, as it is called, is commenced. It will be necessary, moreever, in almost every case, to range the lines between every two of the main stations with long slender ranging poles, as the intervention of hills, fences, trees, buildings, &c., will frequently interrupt the view of even the loftiest station poles that can be obtained; and more especially so, when the main stations are at a great distance, which depends on the magnitude of the survey, and is sometimes as much as ten miles. In measuring these main lines, every fence, road, stream, building, &c., which is passed or crossed must be noted in the

field book, the several crossings and bends being sketched therein, to the latter of which offsets must be taken. Stations must also be left on these main lines, at convenient situations for taking the interior fences, &c., of the survey, and their distances carefully noted in the field book. From and to the stations, thus left, or from and to points near them, secondary lines must be run, as near the interior parts of the survey as possible, the crossings, offsets, and other remarks being made in the field book, as already directed for the measurement of the main lines. These secondary lines will accurately fit between the points from and to which they have been measured, when laid down on the plan; thus forming a net work of small triangles within the four large triangles, into which the survey is divided by the six fundamental lines. This principle of proof is founded on the obvious property of triangles having a common angle always fitting one within another, the common angle of both being coincident. The lines marked with the figures 1 to 6, represent the system in question, those without figures are the secondary lines.

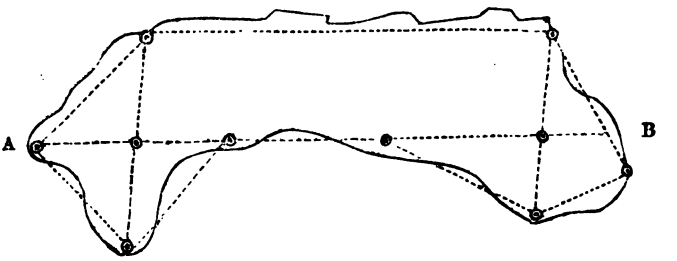


The main lines are numbered with the figures 1, 2, 3, &c., in small circles, as the most convenient method of reference to the field book: the secondary lines must have these numbers continued on them, for the same purpose, but this is not done in the diagram, to avoid confusing it.

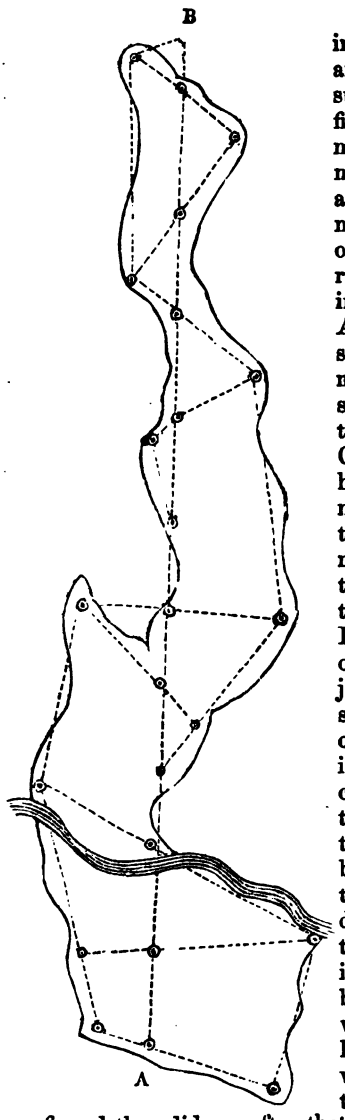
It will be seen that the secondary lines *m n*, *r s* are prolonged beyond the system of main lines to give stability to the parts of the survey that protrudes beyond line 2. It will also be seen that the positions of several straight fences are determined by triple intersections of the main and secondary lines, or by direction points taken in them, as shewn in Prob. IX.

In the author's practice, as a surveyor, he adopted this method, in the survey of the Parish of Tillington, Sussex, with the exception of bringing one of the diagonals to one of the angles of the trapezium, which, of course terminated in one of the side lines; thus giving a system of lines equally perfect, the end of the diagonal being only a few chains from the angle; there being a considerable practical difficulty in laying down this system of lines, where the ground is hilly and woody, as was the case on this occasion.

2. The system of main lines adopted by the author in the survey of the Parish of Woolbedding, in the same county, was the following,



the base line A B being about five miles in length. To this survey, it may be clearly seen that the Tithe Commissioners' system of lines would be not at all adapted; though some surveyors, in compliance with their orders, adopted their system, whatever might be the shape of the survey, thus wasting much valuable time, in running lines over grounds at a great distance, in some parts, from the parish to be surveyed, and incurring the ridicule of both the scientific and the ignorant to boot. Such a course would evidently be required, in using their system in this survey: meanwhile a system of lines adapted to the shape of the survey, and constituting proof among themselves, as those shewn above do, are evidently the best. In this figure the interior fences and secondary lines of the survey are not drawn, as their great number would confuse the student; the author's object being to present a proper system of fundamental lines for the survey in question.



3. The Parish of Lodsworth, in the same county, presents another variety of shape; in the survey of which, the author at first laid the system of fundamental lines shewn in the annexed figure; which had been adopted, had he not, in the meanwhile, obtained the survey of the adjoining parish; both parishes were, therefore, included in one survey. The base line A B of this parish was nearly seven miles in length, its long, narrow, zig-zag shape completely setting aside the *universal* method put forth by the Tithe Commission authorities; who, however, did not insist on their methods being adhered to, as they approved of the author's maps as of the first class, in this and many other cases where their method was not adopted. But such was the obsequiousness or ignorance of the great majority of surveyors that, even in such incongruous cases as the one referred to, they persevered in the Tithe Office rule, in some cases by joining together two, three, or four trapeziums, with their diagonals, and sometimes by making the surveys after their own methods, and then drawing on their maps, the system, or groups of the systems, in question, and making a field-book to correspond thereto; they were thus at liberty to project lines in any direction they chose, without the trouble of measuring them; and many have exultingly confessed they did so, after their maps had received the seal of

the Tithe Commissioners: but how far such behaviour is to be commended, I leave the reader to judge.

Having given the student a great variety of examples, both in small and large surveys, it will now be proper to describe the drawing instruments, required in laying down extensive surveys, as well as the more perfect field instruments, required in railway and other engineering surveys; which are often of a complicated character, as are also private surveys of woodlands and mountainous districts. These subjects will be treated of in Chap. V., where the proper forms of the field books will be given, for extensive surveys of every description.

CHAPTER IV.

DRAWING & SURVEYING INSTRUMENTS.

THE VERNIER SCALE.

The nature of this scale will be best understood from its construction.—To construct a vernier scale, which will enable us to take off a number to three places of figures: divide all the primary divisions into tenths, and number these subdivisions, 1, 2, 3, &c., from the left hand towards the right, throughout the whole extent of the scale. Then take off, with the compasses, eleven of these subdivisions, set the extent off backwards, from the end of the first primary division, and it will reach beyond the beginning of this division, or zero point, a distance equal to one of the subdivisions. Now divide the extent, thus set off, into ten equal parts, marking the divisions on the opposite side of the divided line to the strokes marking the primary divisions and the subdivisions; and number them 1, 2, 3, &c., backwards from right to left. Then, since the extent of eleven subdivisions has been divided into ten equal parts, so that these ten parts exceed by one subdivision the extent of ten subdivisions, each one of these equal parts, or, as it may be called, one division of the vernier scale, exceeds one of the subdivisions by a tenth part of a subdivision, or a hundredth part of a primary division. In our figure the distances between the primary divisions are each one inch, consequently the distances between the subdivisions are each one-tenth of an inch, and the distances of the divisions on the vernier scale are each one-tenth and one-hundredth of an inch.

TO TAKE OFF A GIVEN NUMBER FROM THE VERNIER SCALE.

Example 1. Let the given number be 253.

Increase the first figure 2 by 1, making it 3; because the vernier scale commences at the end of the first primary division, and the primary divisions are measured from this point, and not from the zero point.* The first figure, thus increased, represents 35 of the subdivisions from the zero point, from which the third figure 3 must be subtracted, leaving 32, since three divisions of the vernier scale will contain three of these subdivisions together with three tenths of a subdivision. Now place one point of the compasses upon the third division of the vernier scale, and extend the other point to the 32nd subdivision, or the second division beyond the third primary division, and, laying down the distance between the points of the compasses, it will represent 253, or 25·3, or 2·53, according as the primary divisions are taken for hundredths, tens, or units.

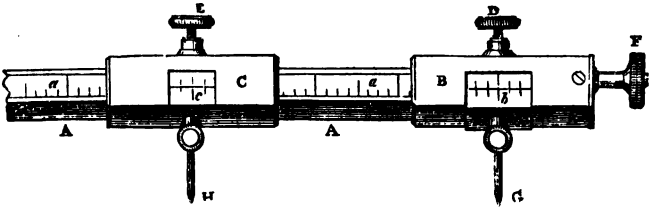
GENERAL RULE.—*To take any number to three places of figures from this venier scale.*—Increase the first figure by one; subtract the third figure from the second, borrowing one from the first increased figure, if necessary, and extend the compasses from the division upon the vernier scale indicated by the third figure, to the subdivision indicated by the number remaining after performing the above subtraction.

DIRECTIONS CONCERNING PLOTTING EXTENSIVE SURVEYS FROM THIS SCALE.

During the time necessarily occupied in plotting an extensive survey, the paper on which the work is drawn, is affected by the different states of humidity of the air, and the parts laid down from the same scale, at different times, will not exactly correspond, unless the scale has been first laid down on the paper itself, and all the distances of the survey have been taken from the scale so laid down, which will always be in the same state of expansion or contraction as the work on the plan. Therefore, for plotting an extensive survey, and accurately filling in the minutiæ, a diagonal, or vernier scale may be advantageously laid upon the paper on which the plan is to be made. A vernier scale is preferable to a diagonal scale, because in the latter it is extremely difficult to draw the diagonals with accuracy, and we have no check upon its errors; while in the former, the uniform manner in which the strokes of one scale separate from those of the other, is strong evidence of the truth of both.

* If the vernier scale were placed to the left of the zero point, a distance less than one primary division could not always be found upon the scale.

THE BEAM COMPASSES.



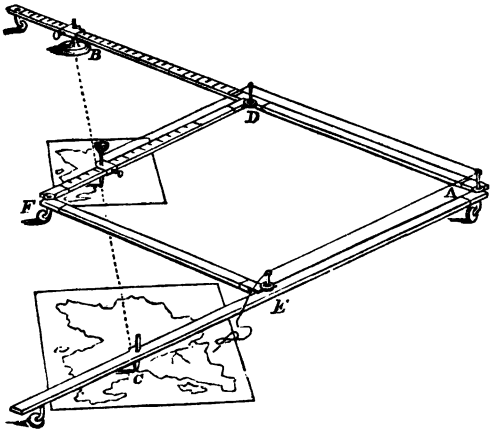
The above engraving represents the instrument, which consists of a beam A A, of any length required, generally made of well-seasoned mahogany. Upon its face is inlaid throughout its whole length, a slip of holly or boxwood *a a*, on which are engraved the divisions or scale, either in feet and decimals or inches and decimals, or such parts of either as may be required. Those made for the use of the Ordinance survey of Ireland, were divided to a scale of chains, 80 of which occupied a length of six inches, which, therefore, represented one mile, six inches to the mile being the scale on which that important survey is plotted; the present survey of the metropolis being plotted to a scale of 60 inches to a mile.* Two brass boxes B and C are adapted to the beam; of which the latter may be moved, by sliding to any part of its length, and its position fixed by tightening the clamp screw E. Connected with the brass boxes are the points of the instrument G and H, which may be made to have any extent of opening by sliding the box C along the beam, the other box B being firmly fixed at one extremity of the beam. The object to be attained by this instrument is the nice adjustment of the points G, H to any definite distance apart. This is accomplished by two verniers, or reading places *b*, *c*, each fixed at the side of an opening in the brass boxes, to which they are attached, and affording the means of minutely subdividing the principal divisions *a a* on the beam, which appear through these openings. D is a clamp screw for a similar purpose to the screw E, that is, to fix the box B, and prevent motion in the point it carries after adjustment. F is a slow motion, by which the point G may be moved any minute quantity for perfecting the setting of the instrument, after it has been otherwise set, as nearly as possible, by the hand alone.

* The first class parish maps of the Tithe Commission survey are to a scale of three chains to one inch, or one mile to 26⅔ inches. Plans of private estates are to various scales, but chiefly varying from 5 to 12 chains to an inch, the smaller scales being chiefly adopted where the estates are large, that the plans may be of a convenient size.

The method of setting the instrument may be understood from the above description of its parts, in conjunction with the following explanation of the method of examining and correcting the adjustment of the vernier *b*, which, like all other mechanical adjustments, will occasionally become deranged. This verification must be performed by means of a detached scale. Suppose, for example, that the beam compass is divided into feet, inches, and tenths, and subdivided by the vernier into hundredths, &c. First set the zero division of the vernier to the zero of the principal divisions on the beam, by means of the slow motion screw F. This must be done with great care. Then slide the box C, with its point H, till the zero on the vernier C exactly coincides with any principal division on the beam, as 12 or 6 inches. To enable us to do this with extreme accuracy, some superior kinds of beam compasses have the box C also furnished with a tangent or slow motion screw, by which the setting of the points of division may be performed with the utmost precision. Lastly, apply the points to a similar detached scale, and, if the adjustment be perfect, the interval of the points G, H will measure on it the distance to which they were set on the beam. If they do not, by ever so small a quantity, the adjustment should be corrected, by turning the screw F, till the points exactly measure that on the detached scale; then, by loosening the little screws which hold the vernier *b* in its place, the position of the vernier may be gradually changed, till its zero coincides with the zero on the beam; and, the screws being now tightened, the adjustment will be complete.

THE PANTAGRAPH.

The pantagraph, used for reducing or enlarging maps, consists of four brass bars A B, A C, D F, and F E: the two longer bars A B, A C are connected by a moveable joint at A: the two shorter bars are connected in like manner with each other at F, and with the longer bars at D and E, and, being equal in length to the portions A D, A E of the longer bars, form with them an accurate parallelogram A D F E, in every position of the instrument. Several ivory castors support the machine parallel to the paper, and allow it to move freely over it in all directions. The arms A B and D F are graduated and marked $\frac{1}{2}$, $\frac{1}{3}$, &c., and have each a sliding index, which can be fixed to any of the divisions by a milled-headed clamping screw, seen in the engraving. The sliding indices have each of them a tube, adapted either to slide on a pin, rising from a heavy circular weight called the fulcrum, or to receive a sliding holder with a pencil or pen, or a blunt tracing point, as may be required.



When the instrument is correctly set, the tracing point, pencil, and fulcrum will be in one straight line, as shewn by the dotted line in the figure. The motions of the tracing point and pencil are then each compounded of two circular motions, one about the fulcrum, and the other about the joints at the ends of the bars, upon which they are respectively placed. The radii of these motions form sides about equal angles of two similar triangles, of which the dotted right line B C, passing through the tracing point, pencil, and fulcrum, forms one side: hence the distances passed over by the tracing point and pencil, in consequence of either of these motions, have the same ratio, and, therefore, the distances passed over in consequence of the combination of the two motions, have also the same ratio, which is that indicated by the setting of the instrument.

The engraving represents the pantagraph in the act of reducing a map to the scale of half the original. For this purpose the sliding indices are first clamped at the divisions on the arms, marked $\frac{1}{2}$; the tracing point is then fixed in the socket at C, over the original map; the pencil is next placed in the tube of the sliding index, on the bar D F, over the paper to receive the copy; and the fulcrum to that at B, on the bar A B. The machine being now ready for use, if the tracing point C be passed delicately and steadily over every line of the map, a true copy, but of one half of the scale of the original, will be marked by the pencil on the paper beneath it. The fine thread, represented as passing from the pencil quite round the instrument to the tracing point C, enables the draughtsman at the tracing point to raise the pencil from the paper, while he passes the

tracer from one part of the original to another, and thus to prevent false lines to be made on the copy. The pencil holder is surmounted by a cup, into which sand or shot may be put, to press the pencil more heavily on the paper, when found necessary.

If the object were to enlarge the map to double its scale, then the tracer must be placed on the arm D F, and the pencil at C; and, if a copy were required of the same scale as the original, then, the sliding indices still remaining at the same divisions on D F and A B, the fulcrum must take the middle station, and the pencil and tracing point those on the exterior bars A B, A C of the instrument.

Though the pantagraph affords the most rapid means of reducing a map or drawing, we cannot recommend its use for enlarging a copy, or even for copying on the same scale, especially when the original drawing is a complicated one; we therefore proceed to mention some of the other means for the attainment of the two latter objects.

To produce a copy of the same size as the original.—Lay a sheet of tracing paper, having its under side rubbed over with powdered black lead, upon the paper intended to receive the copy; the original being then placed over both, the whole may be made to lay steadily by weights placed thereon: the tracing point may now be carefully passed over all the lines of the drawing, with a pressure proportionate to the thickness of the paper; and the paper beneath will receive corresponding marks, forming an exact copy, which is afterwards to be inked in.

NOTE. Copies of maps are also taken for ordinary purposes by laying tracing paper thereon, through which, from its almost transparent thinness, all the lines of the original can be seen, and readily traced with ink, on the tracing paper. Copies, thus obtained, are called tracings.

Another method.—The drawing or map is placed on a large sheet of plate-glass, called a copying glass, and the paper to receive the copy placed over the drawing. The glass is then fixed in such a position as to have a strong light to fall upon it from behind, and to shine through it and both the original drawing and the paper to receive the copy. By this means the lines of the original drawing become visible through the paper to receive the copy, which can be made with precision and ease, without any risk of soiling or injuring the original.

To copy with exactness on a reduced or enlarged scale.—For this purpose we have recourse to the method of squares, by which the most minute details may be copied with accuracy. This perhaps may be best shown by an example. Let figure 1, in the annexed engraving represent a plan of an estate, which

it is required to copy upon a reduced scale of one half. The copy will therefore be half the length and half the breadth, and consequently will occupy but one fourth the space of the original. The subject is the map of an estate, but the process would be precisely the same, if it were an architectural, mechanical, or any other drawing.

Fig. 1.

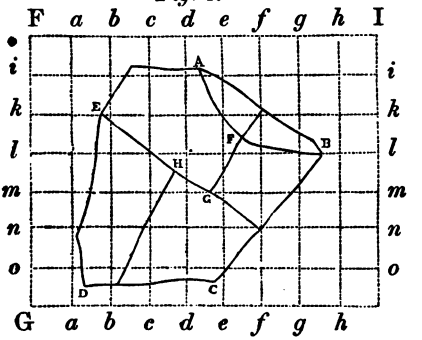
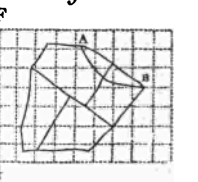


Fig. 2.



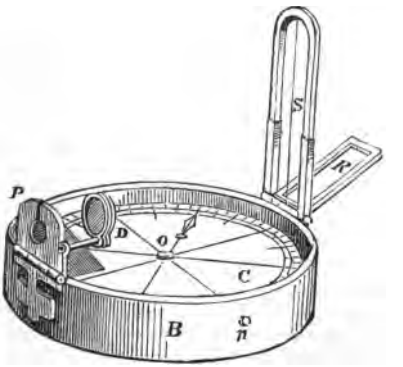
Draw the lines F I, F G at right angles to each other; from the point F towards I and G, set off any number of equal parts, as F *a*, *a b*, *b c*, &c. on the line F I, and F *i*, *i k*, *k l*, &c., on the line F G: from the points in the line F I, draw lines parallel to the other line F G, as *a a*, *b b*, *c c*, &c., and from the points on F G, draw lines parallel to F I, as *i i*, *k k*, *l l*, &c., which being sufficiently extended towards I and G, the whole of the original drawing will be covered with a reticule of small but equal squares. Next draw upon the paper intended for the copy, a similar set of squares, but having each side only one-half the length of the former, as is represented in figure 2. It will now be evident that if the lines AB, BC, CD, &c., figure 1, be drawn in the corresponding squares in figure 2, a correct copy of the original will be produced, and of half the original scale. Commencing then at A, observe where in the original the angle A falls, which is towards the bottom of the square, marked *d e*. In the corresponding square, therefore, of the copy, and in the same proportion towards the left-hand side of it, place the same point in the copy: from thence tracing where the curved line A F crosses the bottom line of that square, which crossing is about two fifths of the width of the square from the left hand corner towards the right, and cross it similarly in the copy. Again, as it crosses the right hand bottom corner in the second square below *d e*, describe it so in the

copy: find the position of the points similarly where it crosses the lines *f f* and *g g*, above the line *l l*, by comparing the distances of such crossings from the nearest corner of a square in the original, and similarly marking the required crossings on the corresponding lines on the copy. Lastly, determine the place of the point B, in the third square below *g h* on the top line; and a line drawn from A in the copy, through these several points to B, will be a correct reduced copy of the original line. Proceed in like manner with every other line on the plan, and its various details, and you will have the plot or drawing, laid down to a small scale, yet bearing all the proportions in itself exactly as the original.

It may appear almost superfluous to remark, that the process of enlarging drawings, by means of squares, is a similar operation to the above, excepting that the points are to be determined on the smaller squares of the original, and transferred to the larger squares of the copy. The process of enlarging, under any circumstances, does not, however, admit of the same accuracy as reducing.

THE PRISMATIC COMPASS.

With this instrument horizontal angles can be observed with great rapidity, and, when used with a tripod stand, with a considerable degree of accuracy; it is, therefore, a useful instrument for filling in the details of an extensive survey, after the principal points have been laid down by means of observations made with the theodolite, hereafter to be described. It was used for this purpose by the gentlemen engaged in making the Ordinance surveys.



C is a compass card, divided usually to every 20′, or third

part of a degree, and having attached to its under side a magnetic needle; *n* is a spring, which, being touched by the finger, acts upon the card and checks its vibrations, so as to bring it sooner to rest, when making an observation. S is the sight vane, having a fine thread stretched along its opening, which is to cut the point to be observed by the instrument. The sight vane is mounted upon a hinge joint, so that it can be turned down flat in the box, when not in use. F is the prism, attached to a plate sliding in a socket, and thus admitting of being raised or lowered at pleasure, and also supplied with a hinge joint, so that it can also be turned down into the box, when not in use. In the plate to which the prism is attached, and which projects beyond the prism, is a narrow slit, forming the sight through which the vision is directed, when making an observation. On looking through this slit, and raising or lowering the prism in its socket, distinct vision of the divisions on the compass card, immediately under the sight-vane, is soon obtained; and these divisions, seen through the prism, all appear, as each is successively brought into coincidence with the thread of the sight-vane by turning the instrument round, as continuations of the thread, which is seen distinctly through the part of the slit that projects beyond the prism.

The method of using the instrument is as follows:—the sight-vane S, and the prism P, being turned up on their hinge joints, as represented in the figure, hold the instrument as nearly in a horizontal position as you can judge, or, if a tripod stand be used, set it as nearly as you can in a horizontal position by moving the legs of the stand, that thence the card may play freely. Raise the prism in its socket till the divisions on the card are seen distinctly through it, and, turning the instrument round, until the object to be observed is seen through the portion of the slit projecting beyond the prism, in exact coincidence with the thread of the sight-vane, bring the card to rest by touching the spring *n*; and then reading at the division upon the card, which appears in coincidence with the prolongation of the thread, gives the magnetic azimuth or bearing of the object observed, or the angle which a straight line, drawn from the eye to the object, makes with the magnetic meridian.* The magnetic azimuth of a second object being

* The magnetic meridian now makes an angle of 24° with the true meridian at London, the north point of the compass being 24° west of the true north point. This angle is called the variation of the compass, and is different at different places, and also at the same place at different times. Since this variation will affect equally, or nearly so, all azimuths observed within a limited extent, and during a limited time, the angles subtended by any two of the

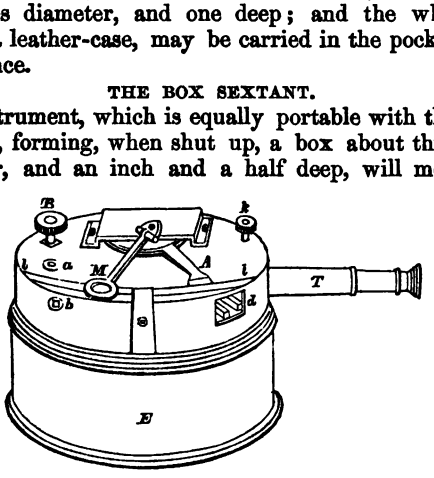
obtained in the same manner, the difference between these two azimuths is the angle subtended by the objects at the place of the eye, and is quite independent of the error in the azimuths, arising from the slit in the prism not being diametrically opposite to the thread of the sight-vane.

For the purpose of taking the bearings of objects much above or below the level of the observer, a mirror B is supplied with the instrument, which slides on and off the sight-vane S, with sufficient friction to remain at any part of the vane that may be desired. It can be put with its face either upwards or downwards, so as to reflect the images of objects considerably above or below the horizontal plane of the eye of the observer. If the instrument be used for obtaining the magnetic azimuth of the sun, the dark glasses D must be interposed between the sun's image and the eye.

There is a stop in the side of the box, not shewn in the figure, by touching which a little lever is raised and the card thrown off its centre; as it always should be, when not in use, or the constant playing of the needle would wear the fine agate point, on which it is balanced, and the sensibility of the instrument would be thereby impaired. The sight-vane and prism being turned down, a cover fits on the box, which is about three inches diameter, and one deep; and the whole being packed in a leather-case, may be carried in the pocket without inconvenience.

THE BOX SEXTANT.

This instrument, which is equally portable with the prismatic compass, forming, when shut up, a box about three inches in diameter, and an inch and a half deep, will measure the



objects observed, being the difference of their azimuths, will not be affected by the variation; and hence the map or plan may be constructed with all the objects in their proper relative positions; but the true meridian must be first laid down on the map, if required, by making allowance for the variation.

actual angle between any two objects to a single minute. It requires no support but the hand, is easily adjusted, and, when once adjusted, seldom requires re-adjusting.

When the Sextant is to be used, the lid E of the box, is taken off and screwed to the bottom, where it makes a convenient handle for holding the instrument; the telescope T, being then drawn out, the instrument appears as shewn in the figure. A is an index arm, having at its extremity a vernier, of which 30 divisions coincide with 29 divisions on the graduated limb *l l*, and the divided spaces on the limb denote each 30 minutes, or half a degree, the angles observed being read off by means of the vernier to a single minute. The index is moved by turning the milled head B, which acts upon a rack and pinion within the box. To the index arm is attached a mirror, called the index glass, which moves with the index arm, and is firmly fixed upon it by the maker, so as to have its plane accurately perpendicular to the plane in which the motion of the index arm takes place, and which is called the plane of the instrument: this plane is evidently the same as the plane of the face of the instrument, or of the graduated limb *l l*. In the line of sight of the telescope is placed a second glass, called the horizon glass, having half its surface silvered, and which must be adjusted that its plane may be perpendicular to the plane of the instrument, and parallel to the plane of the index glass, when the index is at zero. The instrument is provided with two dark glasses, which can be raised or lowered by the little levers seen at *d*, so as to be interposed, when necessary, between the mirrors and any object too bright to be otherwise conveniently observed, as objects in the direction of the sun. The eye end of the telescope is also furnished with a dark glass, to be used when necessary.

To see if the instrument be in perfect adjustment.—Place the dark glass before the eye-end of the telescope, and looking at the sun, and moving the index backwards and forwards a little distance on either side of zero, the sun's reflected image will be seen to pass over the disc, as seen directly through the horizon glass, and if in its passage the reflected image completely covers the direct image, so that one perfect orb is seen, the horizon glass is perpendicular to the plane of the instrument: but, if not, the screw at *a* must be turned by the key *k* till such is the case. The key *k* fits the square heads of both the screws seen at *a* and *b*, and fits into a spare part of the face of the instrument, so as to be at hand when wanted. This adjustment being perfected, bring the reflected image of the sun's

lower limb in exact contact with the direct image of his upper limb, and note the reading of the vernier; then move the index back beyond the zero division of the limb, till the reflected image of the sun's upper limb is in exact contact with the direct image of his lower limb, and, if the zero of the vernier be now exactly as far behind the zero of the limb, as it was at the former reading in front of it, the instrument is in perfect adjustment; but, if not, half the difference is the amount of error, which must be corrected by applying the key *k* to the screw at *b*, and turning it gently till both readings are alike, each being made equal to half the sum of the two readings first obtained. When this adjustment is perfected, if the zeros of the vernier and limb are also made exactly to coincide, the reflected and direct image of the sun will exactly coincide, so as to form but one perfect orb, and the reflected and direct image of any line, sufficiently distant not to be affected by parallax, as the distant horizon, or the top or end of a wall, more than a half a mile distant, will coincide so as to form one unbroken line.

To obtain the angle subtended by two objects in, or nearly in, the same horizontal plane.—Hold the sextant in the left hand, and bring the reflected image of the right hand object into coincidence with the direct image of the left hand object, and the reading of the instrument will give the angle between the two objects.

To obtain the angle subtended by two objects in, or nearly in, the same vertical plane.—Hold the instrument in the right hand, and bring down the reflected image of the upper object, by turning the milled head B, till it exactly coincides with the direct image of the lower object, and the reading of the instrument will give the angle between the two objects.

It will be seldom that the surveyor need pay any attention to the small error arising from parallax, but, should great accuracy be desirable, and one of the objects be distant while the other is near, the parallax will be elliminated by observing the distant object by reflection, and the near one by direct vision, holding the instrument for this purpose with its face downwards, if the distant object be on the left hand. If both objects be near, the reflected image of a distant object, in a direct line with one of the objects, must be brought into coincidence with the direct image of the other object, and the parallax will thus be elliminated.

For the purposes of surveying, the horizontal angles between objects are chiefly required, and the reduction of these angles from the actual oblique angles subtended by the objects, would be a troublesome process. If the angle subtended by two objects be large, and one be not much higher than the other.

the actual angle observed will, however, be a sufficiently near approximation to the horizontal angle required; and if the angle between the two objects be small, the horizontal angle may be obtained, with sufficient accuracy, by taking the difference of the angles observed between each of the objects, and a third object at a considerable angular distance from them. With a little practice the eye will be able to select an object in the same direction as one of the objects, and nearly on a level with the other object, and the angle between this object and the object selected will be the horizontal angle required.

For laying off long offsets, or perpendicular distances from a station line.—The pocket sextant is a most convenient instrument for this purpose: for by setting the index to 90°, and walking along the station line, looking through the horizon glass directly at the further station staff, or any other remarkable object on the station line, any object off the station line will be seen by reflection, when the observer arrives at the point where the perpendicular falls from this object upon the station line, and the distance from this point to the object, being measured, is its perpendicular distance from the station line.

THE OPTICAL SQUARE.

For the purpose of measuring long offsets and perpendiculars, this instrument is now very generally used, which consists of the two glasses of the sextant fixed permanently at an angle of 45°, so that any two objects seen in it, the one by direct vision and the other by reflection, subtend at the place of the observer an angle of 90°.

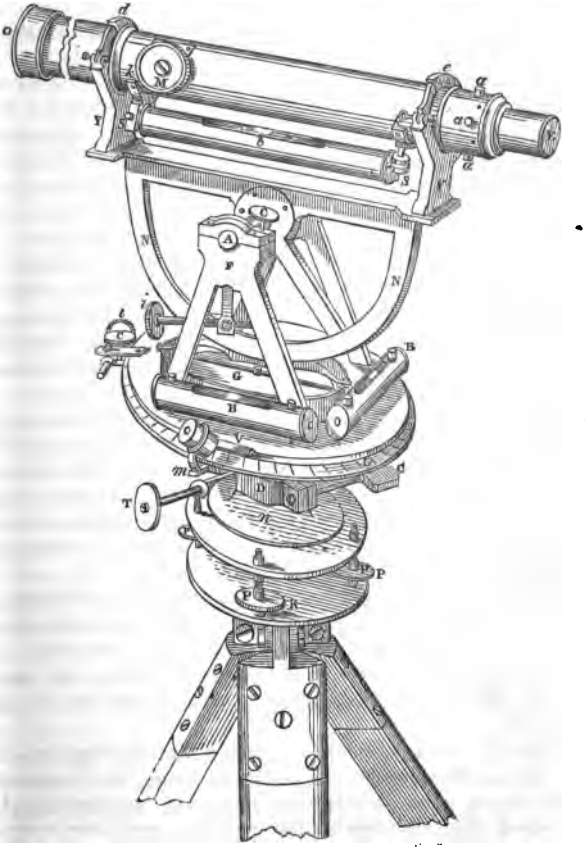
THE THEODOLITE.

The theodolite is the most important instrument used by surveyors, and measures at the same time both the horizontal angles between two objects observed with it, and the angles of elevation of these points from the point of observation.

The instrument may be considered to consist of three parts; the vertical limb for measuring vertical angles, the horizontal limb for measuring horizontal angles, and the parallel plates, in the lower of which is a female screw, adapted to the staff-head, which is connected by brass joints with three mahogany legs, so constructed as to shut together and form one round staff, a very convenient form for portability, and, when opened out, to make a firm stand, be the ground ever so uneven.

The horizontal limb is composed of two circular plates L and V, which fit accurately one upon the other. The lower plate projects beyond the other, and its projecting edge is sloped off, and graduated at every half-degree. The upper plate is called the vernier plate, and has portions of its edge sloped off, so as

to form with the sloped edge of the lower plate continued portions of the same conical surface. These sloped portions of the upper plate are graduated to form the verniers, by which the limb is subdivided to minutes. The five-inch theodolite, represented in the figure, has two such verniers 180° apart. The lower plate of the horizontal limb is attached to a conical axis passing through the upper parallel plate, and terminating in a ball fitting in a socket, upon the lower parallel plate. This axis is hollowed to receive a similar conical axis, ground accurately to fit it, so that the axis of the two cones may be exactly coincident. To the internal axis, the upper, or vernier



THE FIVE INCH THEODOLITE.

plate of the horizontal limb is attached, and thus, while the whole limb can be moved through any horizontal angle required, the upper plate only can also be moved through any desired angle, when the lower plate is fixed by means of the clamping screw C, which tightens the collar D. T is a slow-motion screw, which moves the whole limb through a small space, to adjust it more perfectly, after tightening the collar D by the clamping screw C. There is also a clamping screw *c* for fixing the upper plate to the lower, and a tangent screw *t*, for giving the upper plate a slow motion upon the lower, when so clamped. Two spirit levels B, B are placed upon the horizontal limb, at right angles to each other, and a compass G is also placed upon it, in the centre between the supports F F of the vertical limb.

The vertical limb N, N is graduated on one side at every 30′ minutes, each way from 0 to 90°, and subdivided by the vernier, which is fixed to the compass box, to single minutes. Upon the other side are engraved the number of links to be deducted from each chain, for various angles of inclination, in order to reduce distances measured on ground rising or falling at these angles, to the corresponding horizontal distances. The axis A of this limb must rest in a position truly parallel to the horizontal limb, upon the supports F F, so as to be horizontal when the horizontal limb is set truly level, and the plane of the limb N N must now be perpendicular to its axis. On the top of the vertical limb N N is attached a bar that carries two Ys (so called from their shape), for supporting the telescope, which is secured by two clips *c*, *d*; and underneath the telescope is a spirit level *s s*, attached to it at one end by a joint, and at the other by a capstan-headed screw. The horizontal axis A can be fixed by a clamping screw C; and the vertical limb can then be moved through a small space by the slow-motion screw *i*.

Before commencing observations with this instrument, the following adjustments must be attended to:—

1. *Adjustments of the telescope for parallax and collimation.*
2. *Adjustment of the horizontal limb for setting the levels on the horizontal limb to indicate the verticality of its axis.*
3. *Adjustment of the vertical limb for setting the level beneath the telescope to indicate the horizontality of the line of collimation.*

1. *Parallax and collimation.* Move the object-glass *o* by the screw M, and the eye-glass E with the hand, till distant objects and the cross wires within the telescope, appear clearly defined; and the adjustment for parallax will be completed. Next, direct the telescope to some well-defined object at a

great distance; and see that the intersection of the cross wires cut it accurately; then loose the clips *c*, *d*, that confine the telescope in the Ys, and turn it round on its axis, observing whether the centre of the wires still continue to cut the object, during a whole revolution. If it does, it is in adjustment; if not, the line of collimation, or optical axis of the instrument, is not in the line joining the centres of the eye and object-glasses. *To correct this error*, turn the telescope on its axis, and by means of the four conjugate screws *a*, *a*, &c., that move the cross wires, correct for half the error, alternately loosing one screw and tightening its opposite one, till the cross wires cut the same point of the distant object, during an entire revolution of the telescope round its axis.

2. *Adjustment of the horizontal limb.*—Set the instrument up as level as you can by the eye, by moving the legs of the stand. Tighten the collar D by the clamping screw C, and, unclamping the vernier plate, turn it round till the telescope is directly over two of the parallel plate screws. Bring the bubble *b* of the level *s s*, beneath the telescope, to the centre of its run, by turning the tangent screw *i*. Turn the vernier plate half round, bringing the telescope again over the same pair of parallel plate screws; and, if the bubble of the level be not still in the centre of its run, bring it back to the centre, half-way by turning the parallel plate screws, over which it is placed, and half-way by turning the tangent screw *i*. Repeat this operation till the bubble remains accurately in the centre of its run, in both positions of the telescope; and, then turning the vernier plate round till the telescope is over the other pair of parallel plate screws, bring the bubble again to the centre of its run by these screws. The bubble will now retain its position while the vernier plate is turned completely round, shewing that the internal axis, about which it turns, is completely vertical. The bubbles of the levels on the vernier plate being now, therefore, brought to the centres of their tubes, will be adjusted, and also shew the axis to be vertical. Now, having clamped the vernier plate, loosen the collar D by turning back the screw C, and move the instrument slowly round on the external axis, and, if the bubble of the level *s s* maintain its position during a complete revolution, the external and internal axes are coincident, both being vertical at the same time; but, if the bubble does not maintain its position, it shews that the two parts of the axis have been inaccurately fitted, and the fault can only be remedied by the instrument-maker.

3. *Adjustment of the vertical limb.*—The bubble of the level

s s being in the centre of its tube, reverse the telescope end for end in the Ys, and, if the bubble does not remain in the same position, correct for one half of the error by means of the capstan-headed screw at the end of the level, and for the other half by the vertical tangent screw *i*. Repeat the operation till the result is perfectly satisfactory. Next turn the telescope round a little both to the right and to the left, and, if the bubble does not remain in the centre of its run, the level *s s* must be adjusted laterally by the screw at the other end. This adjustment will probably disturb the first, and the whole operation must be carefully repeated. By means of the small screw, fastening the vernier of the vertical limb to the vernier plate over the compass-box, set the zero of the vernier to the zero of the limb, and the vertical limb will be in perfect adjustment.

NOTE. The adjustments of the theodolite here given are essentially the same as those given by Heather, in his 'Treatise on Mathematical Instruments,' as being adapted to the plate of the instrument, which is taken from his work, though I prefer the concise methods, I have given at page 318 of my Additions to the ninth edition of 'Nesbit's Surveying.'

TO TAKE A HORIZONTAL ANGLE WITH THE THEODOLITE.

The theodolite being assumed to be in proper adjustment, the bubbles in the two levels B B, by a proper opening of the legs of the instrument should be made nearly central, and the plummet suspended beneath it should also hang over the station at which the angle is to be taken: then unclamp the whole instrument by means of the screw C, keeping the other motions clamped, and set the horizontal limb level, as already shewn in the first adjustment. Now clamp the whole instrument and unclamp the vernier plate; set the arrow of the vernier to 360°, or zero, on the lower plate, adjusting the points carefully by the microscope *m*, and the adjusting screw *i*. Again unclamp the whole instrument, turning it to the left of the two stations, between which the angle is to be taken, till the centre of the cross wires in the telescope cut the pole, flag, or other object in the station; then clamp the screw C, and by gently turning the screw T, the most perfect accuracy may be secured. Next unclamp the vernier plate and turn it round till the cross wires cut the object at the second station; then clamp and adjust the vernier plate, as before, and, having obtained perfect accuracy, read off the angle by means of the vernier with the microscope *m*. Lastly, read off the angle, in the same way, with the other vernier, and the mean, or half sum, of the the two angles will be the correct angle.

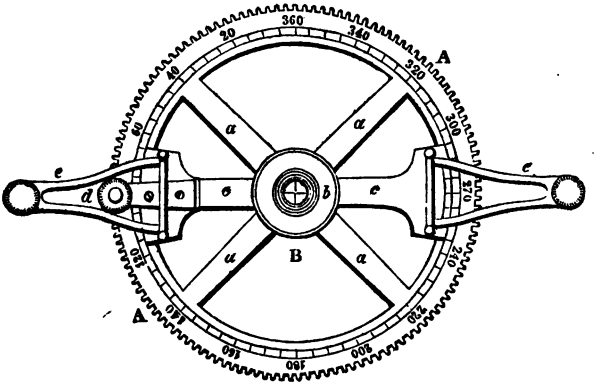
TO TAKE A VERTICAL ANGLE.

Having set the instrument level, as already explained, ob-

serve at the same time whether the zero of the vertical limb coincides with that of its vernier, by the microscope attached thereto. These points being found coincident, raise or depress the telescope, till its optical axis, or cross wires, cut the object required; then clamp, and adjust till perfect accuracy be obtained, when the angle may be read off, which will be an angle of depression, if the arrow of the vernier be between the zero of the vertical circle and the object glass of the telescope; otherwise an angle of elevation.

THE CIRCULAR PROTRACTOR.

This instrument is a complete circle A A, connected with its centre by four radii *a*, *a*, *a*, *a*. The centre is left open and surrounded by a concentric ring, or collar, *b*, which carries two radial bars *c c*. At the extremity of one bar is a pinion *d*, working in a toothed rack quite round the outer circumference of the protractor. To the opposite extremity of the other bar is fixed a vernier, which subdivides the primary divisions on



the protractor to single minutes, and by estimation to 30 seconds. This vernier, as may be readily seen from the engraving, is carried round the protractor by turning the pinion *d*. Upon each radial bar *c*, *c* is placed a branch *e*, *e*, each branch carrying at its extremity a fine steel pricker, whose points are kept above the surface of the paper by springs placed under their supports, which give way when the branches are pressed downwards, and allow the points to make the necessary puncture on the paper. The branches *e*, *e* are attached to the bars *c*, *c* with a joint, which admits of their being folded

backwards over the instrument, when not in use, and for packing in its case. The centre of the instrument is represented by the intersection of two lines, drawn at right angles to each other, on a piece of plate glass, which enables the person using it to place it so that the centre, or intersection of the cross lines, may coincide with any given point on the plan. If the instrument is in correct order, a line connecting the fine pricking points with each other would pass through the centre of the instrument, as shewn by the intersection of the cross lines on the glass; which, it may be observed, are drawn so nearly level with the under surface of the instrument as to do away with any serious amount of parallax, when setting the instrument over a point, from which any angular lines are to be drawn. In using this protractor the vernier should first be set to zero, or the division marked 360, on the divided limb, and then placed on the paper, so that the fine steel points may be on the given line, from whence the angular lines are to be drawn, and that the centre of the instrument may coincide with the given angular point in the same line. This done, press the protractor gently down, which will fix it in position by means of very fine points on its under side. It is now ready to lay off the given angle, or any number of angles, that may be required from the given point, which is done by turning the pinion *d* till the opposite vernier reads the required angle. Then press the branches *e e* gently down, and they will cause their points to make punctures in the paper, at opposite sides of the circle; which being afterwards connected, the line will pass through the given angular point, if the instrument was first correctly set. In this manner, at one setting of the instrument, any proposed number of angles may be laid off from the same point.

PLANNING EXTENSIVE SURVEYS, IMPROVED FORM OF THE FIELD BOOK, &c.

GENERAL DIRECTIONS FOR PLANNING EXTENSIVE SURVEYS.

Provide a sheet of paper, mounted on canvass of the proper size to contain the survey. This may be easily ascertained from the lengths of the longest lines in the survey, and the scale to which it is intended to be laid down. Draw the first line with the proper bearing, which may be determined by the compass of the theodolite, making proper allowance for the variation, as already noticed in the description of surveying instruments. If the survey has been made by the chain only, the bear-

ing may be found by a common pocket compass, or by tying a line to the first or base line in the direction of the sun at 12 o'clock.

If the first or base-line be a very long one, a straight-edge should be provided of equal length, for repeatedly splicing the line, as it is termed, with a short straight-edge will almost invariably throw it out of the right direction, and if the first line be longer than any straight-edge that can be procured, (the author has had lines in maps of his surveys nearly 20 feet in length), in this case, stretch a strong silken thread in the proper direction of the line, and carefully puncture the drawing paper in several places, in the exact direction of the thread, the line may then be correctly drawn, with a short straight-edge, from one point or puncture to another.

The first line being laid down, by one or other of these methods, the stations, crossings, &c., marked thereon, take separately, (in the beam-compasses if the extent of survey require them), the second and third lines, or any other two convenient lines that form a triangle with the line already laid down, and, from the proper stations as centres, describe arcs intersecting each other; thus giving the position of the first triangle, which must now be proved by drawing one or more of the secondary lines from their proper stations therein. In the same manner proceed with the other triangles, formed on the first and other lines, till all the lines are laid down.

If the surveys have been made with the help of an angular instrument, as the theodolite, &c., the angles must be laid off at their proper stations, proving the work as it proceeds.

It may here be proper to observe, that in extensive surveys, the lines measured each day must be laid down, and proved at night, that any error that may have occurred may be corrected before the work has proceeded too far; otherwise the correction will involve greater trouble.—The lines of the survey being thus laid down, all the fences, roads, footpaths, rivers, brooks, ponds, bridges, towns, villages, and detached buildings of every kind, as well as every remarkable object, must next be laid down in pencil, and the fences and outlines of buildings, roads, rivers, &c., drawn with Indian ink; thus finishing what is called the rough plan.

MR. RODHAM'S IMPROVED FORM OF THE FIELD BOOK.

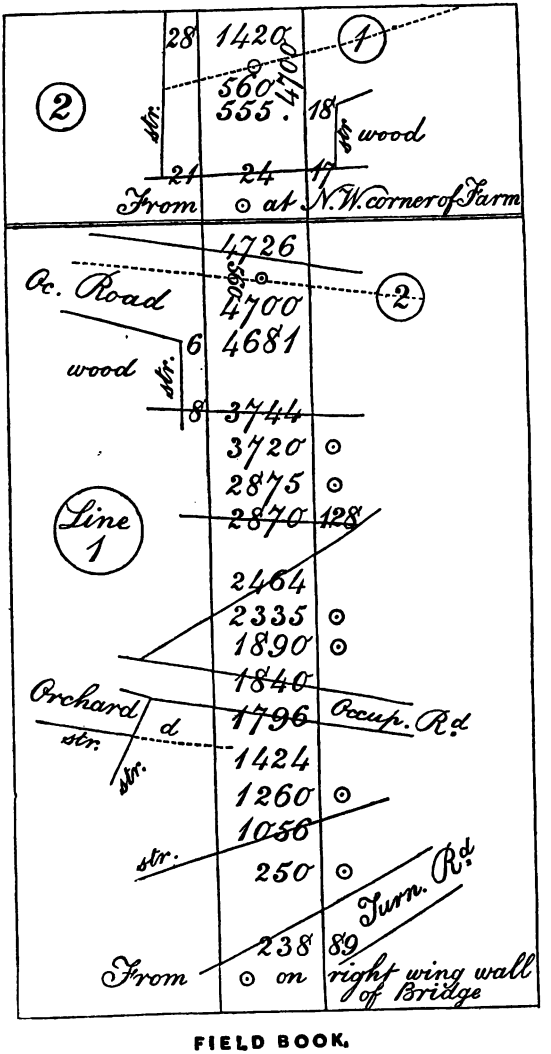
As an example for practice, a plan with the form of the field book, as given by the late Mr. Rodham, of Richmond, Yorkshire, is presented in the plate facing page 82. This form of the field book was first published by Dr. Hutton, above 50 years ago, was then generally adopted, and has not since

been materially improved; the fences and other objects in the survey being sketched therein, on the right and left of the chain line. The author has here departed from the original form by numbering the lines, as being more convenient for reference in large surveys than the method of numbering the stations or marking them with the letters of the alphabet, as recommended by Mr. Rodham.

In the plan referred to, the first line, No. 1, commences at a station in the turnpike road, near N. W. wing wall of the bridge crossing the river Ouse, and is ranged in a north-westerly direction, and so as just to avoid the wood, No. 1; the numbers of this and the other lines being placed in small circles on the lines, to prevent their being mistaken for the numbers of the fields and other enclosures, the same numbers being also placed on the left of the station lines in the field book. Stations are left in line 1 at 250, 1260, 1890, 2335, 2875, 3720, and 4700 links for secondary lines on the right and left of it: this line, having crossed seven fences as shewn in the field book, terminates, in the northern boundary fence of the estate at 4726 links: it will also be seen that the direction of the straight fence on the south of the orchard and homestead, No. 11 is taken, on the left of this line, at 1424 links. The stations referred to, which are afterwards used in the survey, are put down on the right of the station line in the field book, opposite their respective distances; although they might with equal propriety be put immediately above their respective distances in the station-line. Some surveyors prefer this latter method, as the stations do not thus interfere with the space allotted to the offsets on the right of the line: the student is, therefore, at liberty to adopt which method he chooses.

Line 2 commences outside the north-western angle of the estate, crosses the boundary at 24, runs through the station at 4700 in line 1, and terminates at the eastern fence of the turnpike road, stations being left therein at 2360 and 3184, and offsets taken therefrom to the northern boundary fence of the estate, which fence is sketched in the field book.

Line 3 begins from ⊙ at 3184 in line 2, turning to the right thereof. This beginning of line 3 is thus marked in the field book:—"from ⊙ at 3184 ⌐," the symbol ⌐ denoting that line 3 turns to the right of line 2, the symbol ¬ being used when the line turns to the left. This line runs near the turnpike road, for a considerable distance, the two fences of which are taken therefrom by double offsets, as shewn in the field book; and, after crossing the occupation-road between fields 8 and 9, meets the


28
1420
1
560
555.
18
str.
wood
str.
2
21
24
19
From ⊙ at N.W. corner of Farm
4726
Sec.
Oc. Road
4700
2
6
4681
wood
str.
8
3744
3720 ⊙
2875 ⊙
Line 1
2870
128
2464
2335 ⊙
1890 ⊙
1840
Orchard
d
1796
Occup. Rd.
str.
str.
1424
1260 ⊙
1056
str.
250 ⊙
Turn. Rd.
238
89
From ⊙ on right wing wall of Bridge

⊙ at 1260 in line 1, the direction of the straight fence between fields 4 and 8 being taken by a short tie line, as shewn in Prob. IX., Chap. III. The length of this line is 3074 links, and with lines 1 and 2 constitutes a triangle, which may be laid down in the usual way: thus,

Line 4 starts from ⊙ at 250 in line 1 ⌐, and closes on ⊙ at 2097 in line 3, taking the turnpike road by double offsets.

Line 5 starts from ⊙ at 1420 in line 3 ⌐ and meets ⊙ at 2360 in line 2, taking the western fence of field 3 by offsets.

Lines 6 and 7 complete the survey, or filling up, as it is usually called, on the north-eastern side of line 1. It is here proper to observe that the four last lines must all fit, or very nearly so, between their proper stations, when laid down on the plan; otherwise some error or errors has been made, either in the measurements, or in the entries in the field book, or in laying down the lines, which must be corrected before the work proceeds any further.

The next main line 8, starts from the beginning of line 2 ⌐, of which proper notes are made in the field book, and ends on the north bank of the river Ouse, near the south-west corner of the estate, taking the western boundary thereof by means of offsets throughout its whole length, as shewn by the field book.

Line 9 starts from ⊙ at 4713 in line 8 ¬, and meets ⊙ at 459 beyond ⊙ at 250 in line 1, the distance between these stations being measured on finishing line 9, as it was not foreseen that a ⊙ was required for this line, when line 1 was measured, the entry being made as shewn in the field book.

The remainder of the field book is here put down without sketching the fences, &c., which the student can readily do himself for practice from the plan.

		cross 24	Δ	post
	(10)		1250	
	36			
		37	1179	13 farm house
54	out	37	1160	
	buildings	38	1008	
	188			
	36	38	972	12 54
		0	740	43
		8	700	62
		cross 56	608	fence
straight fence		47 40	500	
286		48 44	283	
		From	⊙ at	1890 in (1) ¬

62	2520	
cross straight fence 89	2032	219
	1552	154
	1380	70
cross straight fence	1145	88
buildings { 31 / 38	902	
	1283 ⊙	(10)
	861	
building { 39 / 36, { 40	850	cross fence
	706	
(13)	530	70
cross	501	fence
79	460	
71	100	
cross	92	fence
From	⊙ at	820 in (11) ⌐

To	⊙ at	900 in (8)
	1077	
cross	1032	fence
(12)	900	29
	780	57
	550	31
From	⊙ at	3720 in 1 ¬

To	⊙ at	2875 in (1)
	1526	
	1436	58
	1040	71
(11)	890	98
	820	⊙
	700	69
	560	36
	200	58
cross	74	fence
From	⊙ at	1740 in (8) ¬

cross	2174	fence
	to ⊙ 2560	
	2045	
	1950	
(10) 93	1316	28 fence

	To	⊙ 314	to left of ⊙ at 250 in 1	
		1458		
		870	⊙	R. Ouse 125 links wide.
	cross straight fence	786	31	
(16)		700	24	
		500	18	
		300	58	
		150	106	
	From	⊙ at	1310 in (15)	⌐

	River crosses	1440		
		1360		
		1310	⊙	
		1200	121	
(15)		1025	159	
		880	161	
		750	115	
	cross straight fence 34	610	41	R. Ouse 125 links wide.
	Wood			
	cross straight fence 26	264	28	
		150	91	
	From	⊙ at	1660 in (9)	

	To	⊙ at	4713 in (8)	
		783		
(14)	18	600		
	86	440		
	124	300		
	96	140		
	From	⊙ at	2912 in (13)	⌐

R. Ouse 125 links wide.

	River Ouse	3140	crosses	
		2926		
(13)		2912	⊙	
	900 ⊙	56 ⊙	on (9)	
		2712		

	cross straight		538	fence (wood)
		to	⊙ at	870 in (16)
(17)	R. Ouse 125 links wide.		447	
		36	400	
		22	250	
	128	4	68	}
	Bridge	12	52	} cross
	120	12	10	
		From	⊙ at	0 in (1) ⌐

After the rough plan has been completed, the contents of all the fields, &c., must next be found, by the methods already given in Chaps. II and III. The scale to which the plan referred to is drawn, is 10 chains to one inch; it being drawn to this scale to accommodate it to the size of a page of this work, but for the purpose of finding the contents with accuracy, the scale to which the rough plan is drawn, should not be less than three or four chains to one inch, otherwise the several parts of the plan cannot be measured with accuracy. The fine plan may next be made to any required scale that may be thought most convenient. Fine plans are of various scales, from 5 to 20 chains to an inch, according to the size of the estate, parish, or manor, or to the desire of the proprietor or proprietors. The fine plan is reduced from the rough one by the pantagraph, or by any of the other methods already given, and is accompanied by a book of reference, containing the names and contents of the several fields and other enclosures, the names of the occupiers, and whatever other particulars, concerning the estate, parish, or manor, that may be required. If the estate be a small one, the reference may be put on one side of the plan. The name of the estate, &c., is usually inscribed with large ornamental letters in a vacant corner of the plan, with the scale to which the plan is drawn.

From what has been already explained with reference to the method of laying down plans, keeping the field book, &c., the student will now have no difficulty in conducting such extensive surveys of parishes, estates, manors, as are referred to at the end of Chap. III, whatever be their varieties of shape; since all surveys made with the chain only, are continued systems of triangulation, or the prolongations of all or some of the sides of the fundamental triangles being made the bases of the further extensions, as may be seen at pages 60, 61, where the

lines for the surveys of the parishes of Woolbedding and Lodsworth are given.

Before quiting this subject, it will be proper to remark, that when two, three, or four surveyors are employed in the survey of a large estate, parish, or manor, it is advisable to divide it into four parts by two large base or main lines crossing one another, at any convenient angle, as near the centre of the work as can be judged by roughly examining it. The main lines must next be tied, as near their extremities as convenient, by four other main lines; which, when laid down, will constitute a proof of the basis of the survey, and will form four large triangles, each surveyor surveying one of these large triangles or spaces cut off by the two first lines. The work, in this case, will be as well connected as if it had been done by one surveyor. The system of fundamental lines in such a survey will nearly resemble that proposed by the Tithe Commissioners for their surveys for first class maps of parishes, for the purpose of the commutation of tithes. This system of lines has been already referred to at p. 59, Chap. III.

CHAPTER V.

EXTENSIVE SURVEYS OF VARIOUS KINDS, EITHER WITH OR WITHOUT THE THEODOLITE.

PREVIOUS to undertaking any extensive projects in engineering, as railways, canals, harbours, the improvement of the navigation of rivers, &c., the district or country through which the railway, canal, &c., is proposed to be formed, must first be surveyed, and an accurate map made thereof, to exhibit the surface required to be occupied by it, and shew how far and in what manner the different properties passed over, or near it, may be damaged, if not wholly required by the undertaking, by its cuttings, embankments, &c., if a railway, canal, or harbour; also, by the severance of estates, removal of buildings, diversion of roads, brooks, and watercourses, by the drainage of wells, ponds and watering places, or by filling them up.

When the intended engineering project is laid down on a map of this kind, and, if a railway or canal, the vertical section thereof accompanying it (see Plate III. at the end of the book), the damage done by it may be readily determined, and the diversion of road, watercourses, &c., contrived in the most convenient manner; in order that the several parties affected may be satisfied.

RAILWAY SURVEYS.

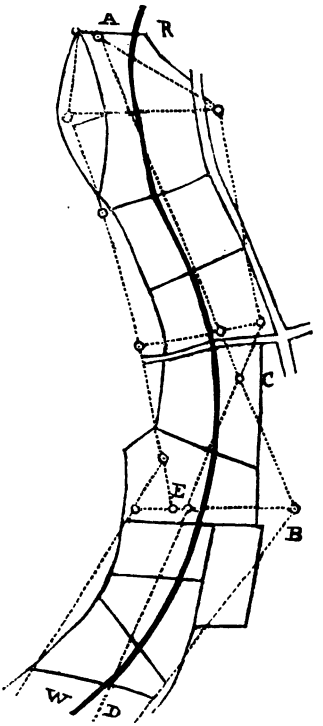
The survey given at the end of Chapter III, viz., that of the Parish of Lodsworth, is adapted to railway surveying, on account of its long and narrow form; and by tieing other base-lines to A B, in that survey, it may be continued to any extent required. But previous to making surveys of this kind, the line of the proposed railway, as far as it can be determined by trial levels, (See Levelling, Part II., Chap. I.) must be roughly delineated, on such a map of the part of the country or district through which it is proposed to pass, as can be procured; (if the railway be in England or Ireland, Ordnance maps are the best for the purpose) with which the superintendent of the survey must be provided that he may know what part or parts of the country is required to be surveyed for the intended project, and in what direction the main lines ought to be taken, which direction must be as near the proposed line of railway as the obstructions arising from woods, rivers, &c., will admit.

Surveys, containing all the required details for this purpose, may be obtained from the first class parish maps, made under the direction of the Tithe Commissioners. These maps may be generally considered as specimens of accuracy, with the exception of a few that were surreptitiously manufactured by disingenuous surveyors; (see page 61) but as it was optional to the parish authorities whether they would have first or second-class maps, they more commonly choose the latter, on account of the expense being less. The great mass of those second class maps, being partly made or compiled from old and incorrect maps, and partly surveyed by unskilful surveyors, are little better than mere sketches of the parishes of which they are put forth as the maps; so much is this the case, that in making surveys for railways, the author, in running a line of little more than a mile in length, has found them to deviate upwards of ten chains from their true position, while fields shewn by the reference to contain about equal contents, were, as shewn by the map, one more than double another in size. These maps are, therefore, utterly worthless for the work in question, for which nothing less than the most accurate surveys are requisite.

When, therefore, a new survey is required to be made, range the first base line, fixing station flags therein at the most convenient points, also other station flags, to the right and left of the main line, in the direction of fences, roads, rivers, &c., except where their are natural marks that will answer the same purpose as the flags. The measurement of the base line,

and the filling up of the survey on the right and left may then proceed, in the same manner, as already shewn in the narrow part of the Lodsworth survey just referred to. When the first base or main line begins to leave the direction of the proposed line of railway, a second main line must then be set out, from a station 10 or 15 chains short of the extremity of the first main line, that the two main lines may thus be effectually tied to each other; after which the survey may be continued along any number of main lines. Sometimes obstructions prevent the effectual tieing together of the main lines, in this case the theodolite must be used, as shall be shewn hereafter. The width of the survey should be from 5 to 20 or 30 chains; the greater widths being where it has not yet been settled where the line of railway shall pass, or where there are curves therein, or some other engineering difficulty.

The annexed figure represents a survey of this kind; in which the thick, curved line R W, is the projected railway; A B the first base line, which at B begins to leave the direction of the line of the railway. At ⊙ C in the first main line another main line C D is set out, crossing and recrossing the railway. The line C D is connected with A B by the tie line B E. In the same manner the next main line may be connected with C D, and the survey conducted thus to any extent or in any direction. The filling up of the chief parts of the survey is shewn by the dotted lines on both sides of the main lines.



ENGINEERING AND OTHER SURVEYS BY THE THEODOLITE.

The use of the theodolite is either preferable, or absolutely necessary, both in engineering and other surveys; the chief cases of which are the following:—

1. In the surveys of woods, lakes, harbours, &c., by ranging a system of lines round them, and connecting them by taking the angles at their several points of junction.

2. In the surveys of winding roads, rivers, &c., especially where they are obstructed by woodlands, buildings, &c., on both sides, by taking the angles, as in the first case.

3. In the surveys of the streets, of parts or the whole of cities, and large towns.

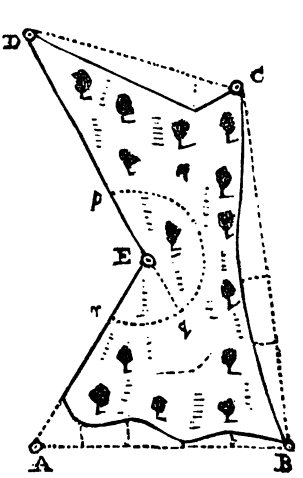
4. In determining the positions and distances of several stations, in an extensive engineering or other survey, by taking angles to them from two or more stations, the distances of which are known.

5. In railway surveys, &c., by connecting the main line with angles, as in the first and second cases, by determining the inaccessible parts that may occur, as in the fourth case; in ranging a long line over high summits. This case frequently embracing all the four preceding ones.

Problem I.

TO SURVEY WOODS, LAKES, HARBOURS, &c., BY THE THEODOLITE.

1. Required the plan and area of the wood represented in the following figure, the field notes, adapted to the theodolite, being given.



Fix station flags round the wood, so that the lines compassing it may be as near it as possible, for the convenience of

taking the offsets; the stations at the same time being made on proper ground for fixing the theodolite. Let A B C D E be the stations, and the field book as below.

	to ⊙ C		From ⊙ E	58° 23′	to ⊙ B
0	2678			to ⊙ A	
101	1400			1793	
119	800		0	1350	
0	000		0	000	
From ⊙ A	81° 29′	to ⊙ C	From ⊙ D	241° 38′	to ⊙ A
	⊙ B ⌝			⊙ E ⌜	
	to ⊙ B			to ⊙ E	
0	2302		0	1790	
99	1800		0	000	
0	1100		From ⊙ C	46° 51′	to ⊙ E
202	600			⊙ D ⌝	
218	320			to ⊙ D	
perpr. to corner	225		0	1898	
Begin at	⊙ A	go E.	237	200	
			0	000	
			From ⊙ B	111° 39′	to ⊙ D
				⊙ C ⌝	

THE METHOD OF TAKING THE ANGLES, &c.

It will be seen, from the fieldnotes, that the line A B is first measured, as a base for the plan. The first angle ABC is then taken, which is found to be 80° 29′, and shews the direction of the second line B C. In taking this angle, the theodolite is fixed directly over ⊙ B; the two zeros of the horizontal plates being then clamped together, the object glass of the telescope is directed to the flag or other mark at ⊙ A, and the whole instrument clamped; the upper plate is now unclamped, and the telescope directed to the flag at ⊙ C, when the angle A B C is found to be 81° 29′, as shewn in the field notes. In a similar manner, the angles at stations C and D are respectively found to be 111° 39′ and 46° 51′, the three lines B C, C D, D E each bending respectively to the left of the line preceding it. The angle at ⊙ E is found to be 241° 38′, which, being greater than 180°, that is, greater than the semicircle *p q*, shews that the line E A turns to the right. Finally, the angle at ⊙ A is found to be 58° 23′, shewing that the line A B turns to the left of E A. Thus the magnitude of the angle shews whether the

new line inclines to the right or the left of the old one, *the new line turning to the left of the old one, when the angle is less than* 180°, *and to the right when greater, the zero of the instrument being always directed to the commencement of the old line:* therefore, the bearing of all the lines, except the first, may be omitted in the field notes; the remainder of which, being similar to those already given, need no further explanation.

PLANNING AND PROVING THE WORK.

Draw the base line A B in the given direction, indefinitely, and lay off the given length 2302 links thereon. Place the centre of the protractor at ⊙ B, with its straight side close against A B, and prick off 81° 29′ from its end towards A; then, through ⊙ B and the protractor-mark, draw B C, making it the given length 2678. Lay down similarly the two following lines with their angles C and D. The angle at ⊙ E is 241° 38′, therefore E A must turn to the right, and the angle to be laid off at E is 360° − 241° 38′ = 118° 22′; or, if a circular protractor be used, the whole angle may be laid off at once, and E A being drawn, must reach to ⊙ A, where the survey began, or so very near to it that the error may be considered immaterial; but if it do not reach to ⊙ A, by a considerable distance, there has been an error either in taking the angles or measuring the lines. But since *the sum of all the interior angles of a polygon is equal to twice as many right angles as the figure has sides, lessened by four right angles*, and since the given figure has five sides, the sum of all its five interior angles will be = 5 × 2 − 4 = 6 right angles = 90° × 6 = 540°. This will be found to result by adding all the angles of the figure, as below.

Angle at	B =	81°	29′
———	C =	111	39
———	D =	46	51
———	E =	241	38
———	A =	58	23
Proof, as respects the angles		540°	0′

The above result shews that the angles have been accurately taken; if, therefore, the work do not close, that is, if E A does not reach to ⊙ A, the error is in the measuring of some of the lines, or in making a wrong entry in the field notes, which may now be readily detected, if required.

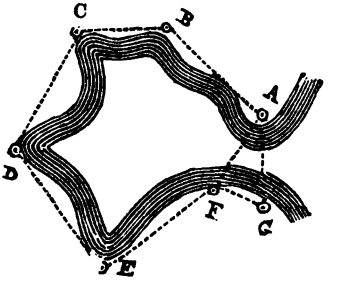
When the work is planned, the content will be found to be 37*a*. 3*r*. 4*p*.

It will be seen that the above survey might have been effected by the method given in Prob. VIII., Chap. III.; but it is here assumed that the lines compassing the wood cannot be prolonged for the purpose of measuring tie lines, on account of obstructions, as is often the case.

Lakes, Meres, and large Ponds are surveyed and planned in the same manner as the wood just given.

2. The following figure represents a gulf or inlet of the sea, the survey of which is required to adapt it for the purposes of a harbour for ships.

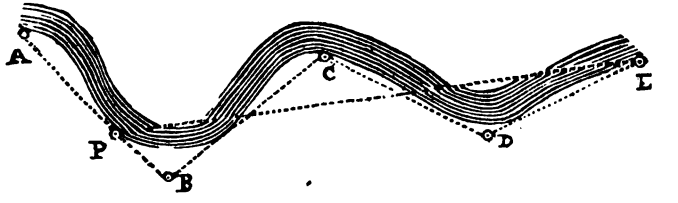
The coast, here shewn, is the boundary of high water; the survey begins at ⊙ A, station-flags being fixed at B, C, D, E, F, and G, and angles taken at A to F and G. The line B A, being first prolonged backwards to high water mark is then measured to B, and the angle A B C taken. Similarly all the succeeding lines are measured, and the angles taken; also at F and G the angles are taken to ⊙ A, and the line F G prolonged to high water mark, all the offsets being taken as the work proceeds. The figure may now be laid down, precisely as in the last example, the magnitude of the angles shewing the directions of the lines, and the lines A F, A G, which could not be measured on account of the great width of the entrance of the harbour, proving the work by means of the angles, taken at A, and F, and G.



Problem II.

THE SURVEYS OF ROADS AND RIVERS.

1. The following figure represents a river, the survey and plan of which is required, for the purpose of improving its navigation, making locks, &c.



Station flags being set up, at or near the principal windings of the river, as at A, B, C, D, E, the line A B is measured, and the offsets taken to the nearest shore of the river, its widths, if very great, being determined by Prob. IV., Chap. III.; or by throwing a leaden ball across the river, with a slender cord attached thereto at one end, and holding the other end in the hand, and then drawing it back, and measuring the length of cord, required to reach the opposite shore. This last method is impracticable where the width of the river is very great. A flag is left at ⊙ P in A B, where the sight, in the general direction of the river, is unobstructed for a considerable distance. The measuring of A B being now finished, the angle at B is taken, which, being less than 180°, shews that B C turns to the left. On measuring to C the angle there is found to be greater than 180°, shewing that C D turns to the right; and thus the work proceeds to ⊙ E, where an angle is taken to the now-distant flag at P. This last angle will prove the accuracy of the work when laid down.

If the width of the river be very great and unequal, a similar system of lines must be used on the opposite shore; otherwise, a correct map cannot be obtained, the two systems of lines being connected by finding occasional widths of the river, as already stated; thus the line B C may be prolonged across the river to connect another system of lines, if required.

2. If a *road* be represented by the winding figure, in the last example, it may be surveyed precisely in the same manner, excepting that it would be more convenient to have the system of lines A B C D E upon the road, instead of the side of it, that the offsets may be readily taken to the right and left of the several lines, recollecting to leave a flag, or some other prominent object, in or near the first line, as at ⊙ P, in the last figure, to which an angle may be taken, after the survey has proceeded a considerable distance, to prove the work.

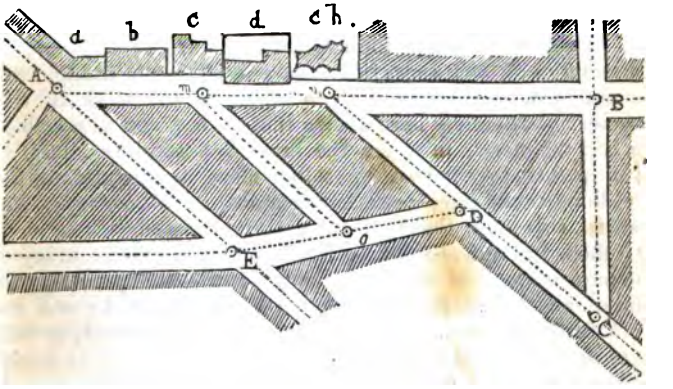
The method of planning either a river or road will be sufficiently clear, from the first example in Prob. I.

The map of a river or road being thus obtained, if the area be required it may be readily found by the methods previously given, or, by taking widths at the end of every chain, estimated along the middle of the river or road, and taking a mean of the widths, which, multiplied by the length, will give the area sufficiently correct for most cases.

Problem III.

THE SURVEY OF A PART OR THE WHOLE OF A CITY OR LARGE TOWN.

Commence the survey at the meeting of three or more of the principal streets, through which the longest prospects can be obtained, for the purpose of laying out the main lines. Having then selected a proper station, fix the theodolite thereon, making a line in one of the principal streets a base line, and directing it to some prominent or well-defined point, as to a projecting corner of a house, or to a lamp, or railing-post, or to the right or left side of a conspicuous door or window, which point must be described in the note book, that it may be remembered. Angles must first be taken between the base line and the other lines, diverging from the first station, defining carefully the directions of these lines, as in the case of the base line. This done, measure these lines with the chain, taking offsets to all corners of streets, bendings, or windings, and to all remarkable objects, as churches, halls, colleges, eminent buildings, &c.; also, defining the extent of buildings belonging to each separate owner, or joint-owners, especially if such buildings are required to be taken down for engineering purposes, or for improvements: at the same time, recollecting to



leave stations opposite the ends of the streets to the right and left, and to take the angles of their directions. This operation must be repeated on the other main lines, till the survey is completed.

Thus, having fixed the theodolite at A, take the angles of lines meeting there, referring them to the base line A B, that

the magnitude of the angle may shew their direction: then measure A B, taking offsets to the buildings of different proprietors, as to the buildings marked *a*, *b*, *c*, &c., on which the dimensions of their several parts, yards, &c., must be put in the note book, that they may be accurately mapped, preparatory to their valuation, if required to be taken down for engineering purposes, or for parish rating; stations being left at *m* and *n*, for the lines in the streets on the right, and the angles of their directions taken; strong iron pins, driven into the crevices of the pavement, being the usual station marks in large towns. The measurement thus proceeds to ⊙ B, where the angles of the streets diverging from that are now taken. The line B C is next measured in like manner, and the angles taken at ⊙ C; after which the measurement proceeds to ⊙ *n*, in the base A B; thus constituting the triangle *n* B C, a station being left at D in C *n*. From ⊙ D the survey proceeds to ⊙ E, and from thence to ⊙ A, where the work commenced, a station having been left in the line D E at *o* for the line to ⊙ *m*. In this manner the survey may be continued to any required extent.

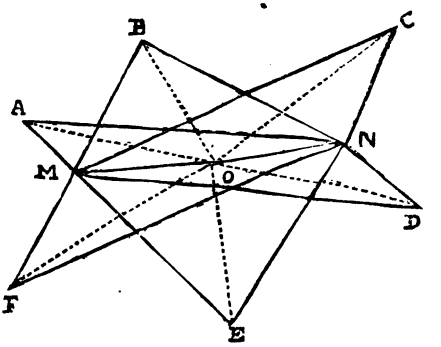
This survey, so far as it has been here shewn, may be plotted independent of the angles taken with the theodolite, by first laying down the triangle *n* B C, and then determining the position of ⊙ E by intersection from stations A and D, when the line *n o* will prove the work. But it rarely happens, in the practice of town surveying, that a fundamental triangle can be obtained, sufficiently large to lay down the work in this manner; it is merely here shewn that such a case is possible; for had the street, in which the line C *n* is measured, been so bent as not to admit a right line along it, the use of the theodolite would have been indispensable in this survey. Assuming, therefore, that C *n* is not a right line, but bent at ⊙ D; then, in the five-sided figure A B C D E, the accuracy of the measurement of the angles may be proved, by taking the sum of all the interior angles as in Ex. 1., Prob. I.; and the work further proved, by the closing of the lines at ⊙ A, as well as at the several other stations.

The form of the field (or rather town) notes in this case, is the same as those already given in the field book, Plate p. 82, excepting that the entries must be made at a sufficient distance apart, that sketches of buildings, yards, gardens, &c., may be clearly made, with the measures of their several parts put on them in links, and the angles must be noted in the same manner as in the example of the wood, Prob. I.

Problem IV.

TO DETERMINE THE POSITIONS OF SEVERAL DISTANT POINTS, OBJECTS, OR STATIONS, BY TAKING ANGLES AT TWO STATIONS AT THE ENDS OF A GIVEN LINE.

Let A, B, C, D, E, F be six stations, the positions of which are to be found. Measure a line M N on level ground, and such that all the stations may be seen from M and N, and at each of the stations M and N, take angles with the theodolite to all the stations; its zero, in taking the angles at each station, being directed to the opposite end of the given line, that the magnitude of the angle may determine the direction of each line of sight to the distant stations. The line M N being then laid down, and the angles taken at its extremities, the intersections of the lines of sight M A, M B, &c., and of N A, N B, &c., will determine the positions of the several distant points A, B, &c., from whence their distances may be found, and made base lines for the further extension of the survey, if required.



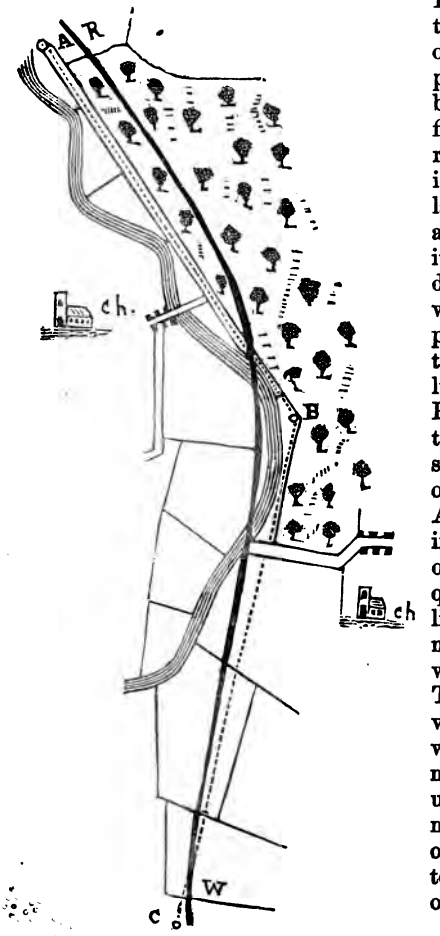
The accuracy of the work may be proved by taking a ⊙ *o*, at any point in M N, and taking from thence angles to all the distant stations; which angles, being laid down, their lines of sight will pass through the intersections A, B, &c., if the work has been correctly done.

When all the distant stations or objects cannot be seen from two given stations, then three stations may be taken, or as many as are necessary; connecting the stations thus used, by measuring their distances, and proving their positions by other lines, or by angles: the zero of the theodolite in taking the angles to the distant stations being always directed to the last given station. Moreover, the angles to all remarkable objects that can be seen from two given stations may be taken at the same time; thus may their positions also be determined by the intersection of two or more lines of sight. In this manner,

very extensive surveys may be effected, occasionally checking the accuracy of the work by measuring the distances of some of the distant stations, where it can be the most conveniently done.

TO SURVEY A DISTRICT WHERE OBSTRUCTIONS OCCUR, FOR A RAILWAY, CANAL, ETC.

In the figure RW is a portion of a projected railway, and AB, B C the main lines to survey that portion of the district occupied by or affected by the railway. The first main line A B runs along a road, till it is obstructed by a large wood on the right, a brook being close to its left, and, since the direction of the railway changes near the point of obstruction to the left, a new main line B C is taken. Here the use of the theodolite is indispensable to take the angle of the two main lines A B, B C at B, as it is impossible, on account of the obstructions in question, to obtain tie lines to connect the main lines, excepting with great trouble. The parts of the survey adjoining the wood and the brook may be filled up in the usual way, while angles must be taken from two or more stations, to determine the positions of churches, and other



remarkable objects on the right and left of the railway, by intersection; it being unnecessary, at the same time, to measure their distances, especially if those distances from the railway be considerable, their positions being only required to shew more clearly the locality of the proposed railway to those interested therein. Also, at the same time, the angles of fences, or other parts of the actual survey that may be in the same direction of the churches, &c., may be conveniently taken, which will thus save the trouble of measuring tie lines. As the railway again leaves the direction of the main line B C at C, a third main line will be required (this line is not shewn in the figure); which may be either tied to B C, or its angle of inclination thereto be taken with the theodolite, according to convenience. The filling up of the part of the survey to the right and left of the part of B C, towards C may be readily effected by the help of the prismatic compass, or box sextant, on account of the fences being straight, and the fields large; the angles which these straight fences make with B C, being taken, and the lengths of the fences measured to the meetings of other straight fences.

CHAPTER VI.

SECTION I.

THE VARIOUS METHODS OF LAYING OUT ANY GIVEN QUANTITY OF LAND IN ANY PROPOSED REGULAR OR IRREGULAR FORM; OF SEPARATING ANY REQUIRED QUANTITY OF LAND FROM AN ENCLOSURE OR COMMON; AND OF DIVIDING LAND OF VARIABLE VALUE AMONG SEVERAL CLAIMANTS, IN PROPORTION TO THEIR RESPECTIVE CLAIMS.

PRELIMINARY PROBLEMS.

PROBLEM I.

To lay out a given quantity of land in the form of a square.

RULE.—Reduce the given quantity to square links, by the table of square measure page 15; and the square root of the result is a side of the required square, in links.

EXAMPLE.

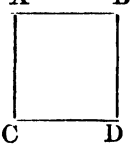
1. Lay out 2*a*. 1*r*. 5½*p*. in the form of a square.

2*a*. = 200,000 square links.
1*r*. = 25,000
5½*p*. = 3,437

228,437 (478 { links nearly, = a side of the square A B D C.
16

87)684
609

948)7537
7584



2. Required the side of a square, that shall contain 3*a*. 3*r*. 28*p*. *Ans*. 626½ links.

NOTE. A given quantity of land is sometimes required to be laid out in the form of a square for pleasure grounds, &c.

PROBLEM II.

To lay out any proposed quantity of land in the form of a rectangle, the length of which is given.

RULE.—Divide the square links in the quantity of land by the given length, and the quotient will be the breadth of the required rectangle.—If the dimensions are required in feet, the given area must be reduced to square feet.

EXAMPLE.

1. Lay out 4*a*. 0*r*. 12*p*. in a rectangular form, the length of which shall be 812 links.

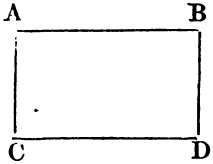
4*a*. 0*r*. 12*p*. = 407,500 square links, and

812) 407500 (501·8 links = A C, the required breadth of the rectangle A B D C.
4060

1500
812

6880
6496

384



2. The breadth of a rectangular piece of ground 140 feet, required its length, when its area is 2 acres.

1 acre = 43560 square feet

2

14·0 { 2 | 8712·0 square feet in 2 acres
       7 | 4356

622⅞ feet, the length required.

3. The length of a rectangular pleasure ground is proposed to be 1000 feet, required its breadth so that it contain 12½ acres. *Ans.* 544½ feet.

NOTE. Besides the extensive use of this Problem in the division of commons, &c., it is also used to determine the length of frontage of a given quantity of building ground, when its depth or breadth is given; or, to determine the depth, when the length of frontage is given.

PROBLEM III.

To lay out a given quantity of land in the form of a rectangle, the length of which shall have a given proportion to its breadth: that is, the length shall be to the breadth as 3 *to* 1, *or as* 4 *to* 3, *or as* 5 *to* 2, *&c.*

RULE.*—Divide the given area by the product of the two terms of the proportion, and the square root of the quotient, multiplied separately by the terms of the proportion, will give the required length and breadth.

EXAMPLE.

1. Lay out 3 acres of land in the form of a rectangle, the length of which shall be to its breadth as 3 to 2.

3 × 2 = 6)300000 square links in 3 acres

50000 (223·6
4

42)100
84

443)1600
1329

4466)27100
26796

..304

links.
223·6 × 3 = 670·8 length.
223·6 × 2 = 447·2 breadth.

* Let mx and nx be the length and breadth of a rectangle, which are in the ratio of $m : n$, and a its area; then $mx \times nx = mnx^2 = a$, or $x^2 = \frac{a}{mn}$, or $x = \sqrt{\frac{a}{mn}}$. Q. E. I.

2. Lay out $6\frac{1}{4}$ acres in a rectangular form, the ratio of whose length and breadth shall be as 5 : 2.

$5 \times 2 = 10)625000$

$\sqrt{62500} = 250.$

links.

$250 \times 5 = 1250 =$ length.

$250 \times 2 = 500 =$ breadth.

3. It is required to lay out a rectangular fish pond, the length of which shall be four times its breadth, and its area 10 acres.

Ans. Length 2000 and breadth 500 links.

NOTE. Proportions of this kind are frequently required in laying out ornamental ground, ponds, &c., in rectangular forms.

PROBLEM IV.

To lay out a triangle, of a given area and a given base, one of the sides of which shall have a given position.

RULE 1.—Divide twice the area by the base for the perpendicular, which erect at any point in the base, and at the extremity of the perpendicular and at right angles to it, range a line till it meet the side given in position, which point of meeting is the vertex of the triangle required.

EXAMPLE.

1. It is required to lay out a triangle A B C to contain 2*a*. 2*r*. 16*p*. on a base A B of 1200 links, the position of A H, of which the side A C is a part, being given.

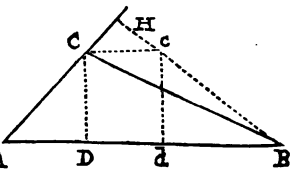
By Rule 1.

2*a*. 2*r*. 16*p*. $= 260000$ links

2

$1200)520000$

$433\frac{1}{3}$ links $=$ CD



On any point *d* of the given base A B $= 1200$ links, erect the perpendicular $d\,c = 433\frac{1}{3}$ links; range *c* C perpendicular to *d c* till it meet A H in C; then, B C being joined, A B C is the triangle required. For let C D be drawn perpendicular to A B, then, by the nature of the construction, C D $= c\,d$. Q. E. I.

RULE 2.—Measure H B perpendicular to A H, the line given in position, to the end B of the given base, and divide twice the given area by H B, and the quotient is A C, which, being measured off, will give the point C.

2. Solve the last example by Rule 2, the perpendicular B H being 860 links.

$$\frac{260000 \times 2}{8{\cdot}60} = 604{\cdot}65 \text{ links} = \text{A C}.$$

3. Required the perpendicular of a triangle, containing 26*a*. 2*r*. 28*p*., its given base being 3112 links.

Ans. 1730½ links, nearly.

NOTE. This Problem is extensively used in the division of lands, commons, &c., as shall be hereafter shewn.

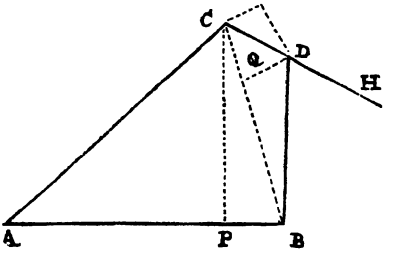
PROBLEM V.

To lay out a trapezium of given area, the positions and lengths of two of its adjoining sides being given, and also the position of its third side.

RULE.—Join the ends of the two adjacent sides, thus forming a triangle, the area of which must be found, which area must be subtracted from the given area, and the remaining area must be laid out in the form of a triangle, on the line joining the two given sides, as base, by Prob. IV.

EXAMPLE.

1. Let the given area of the trapezium A B D C be 6 acres, A B, A C the sides that are given in length and position, and C H the position of the third side. Join B C; measure the perpendicular C P, which is found = 840 links, and C B = 880 links, A B being = 1200 links; whence the area of the triangle A B C = 600 × 840 = 504000 square links; and 600000 — 504000 = 96000 = area of the triangle B C D; therefore 96000 × 2 ÷ 880 = 218$\frac{2}{11}$ links = perpendicular D Q; which, it will be seen, is applied as in Prob. IV.; thus constituting the required trapezium A B D C. C D may also be found as by Rule 2, Prob. IV.



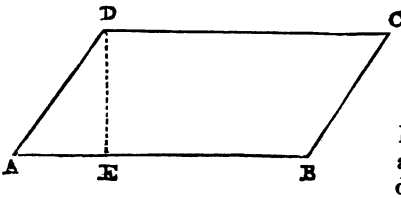
NOTE. This Problem is also much used in the division of land, commons, &c.

PROBLEM VI.

To lay out a rhomboid of given area, the lengths of its sides being given.

RULE.—Divide the area by the longer side, subtract the square of the quotient from the square of the shorter side, and the square root of the remainder is the distance of the perpendicular from the end of the larger side, which perpendicular, being made equal to the quotient first found, gives the breadth of the rhomboid.

EXAMPLE.



1. The area of a rhomboid is required to be 6 acres, and its longer and shorter sides to be respectively 1200 and 625 links, required the place and length of its perpendicular.

$12{\cdot}00)600000$

500 = perpr. = D E

$625^2 = 390625$

$500^2 = 250000$

$\sqrt{.140625} = 375$ links = A E.

2. Lay out the figure when its sides are each 4 chains, that is, when it is a rhombus, its given area being 1*a*. 1*r*. 24*p*.

Ans. 350 links = perpendicular, and its distance 193·6 links.

Problem VII.

To lay out a proposed quanti'y of land in a regular polygon.

Rule.—Divide the area of the proposed polygon, by its corresponding area in the following table, and the square root of the result will be the length of its sides. Multiply the side just found by the corresponding radius, in the column marked radii, and the result will be the radius of the circle that circumscribes the required polygon.

A table of regular polygons, with their names, areas, and radii of their circumscribing circles, the sides of the polygons being unity.

No. Sides.	Names.	Areas.	Radii.
3	Triangle.....	0.433	0.577
4	Square	1.	0.707
5	Pentagon	1.72	0.851
6	Hexagon.....	2.598	1.
7	Heptagon	3.634	1.152
8	Octagon.....	4.828	1.306
9	Nonagon	6.182	1.462
10	Decagon.....	7.694	1.619
11	Undecagon.....	9.365	1.775
12	Duodecagon	11.196	1.932

Note. If the square of the side of a polygon be multiplied by its corresponding area in the preceding table, the product will be the area of the polygon.

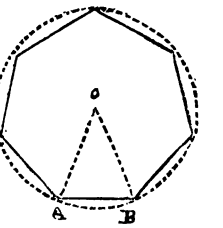
EXAMPLE.

1. Lay out an acre of land in a regular heptagon.

$$\text{Here } \sqrt{\frac{\cdot 100000}{3\cdot 634}} = 165\cdot 88 \text{ links} = \text{A B}.$$

$$\text{Also, } 165{,}88 \times 1\cdot 152 = 191\cdot 09 \text{ links} = \text{A O}.$$

Construction of the heptagon on the ground.—Provide a strong cord, equal in length to the radius of the circumscribing circle of the heptagon, which in this case is 191·09 links, or very nearly $191\frac{1}{10}$ links; fix one of its ends at *o*, as a centre, and tie the other end to a pointed arrow; stretch the cord to A, and with the radius O A describe the dotted circle, keeping the arrow vertical, and making a visible circular mark with it. Then take the distance A B = 165·88 links on the cord, which, being applied seven times within the circle, will just reach round the circumference, thus forming the heptagon required.



2. Lay out 4 acres of land in a hexagon.

Ans. 392·38 links = radius of circumscribing circle and side of the hexagon, for both are equal in this figure.

Problem VIII.

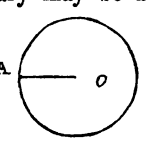
To lay out a given quantity of land in a circular form.

Rule.—Divide the given area by ·7854, and half the square root of the quotient will be the radius of the circle.

EXAMPLE.

1. Lay out 4 acres of land so that its boundary may be a circle.

$$\text{Here } \frac{1}{2}\sqrt{\frac{400000}{\cdot 7854}} = 356\cdot 8 \text{ links} = \text{A O}.$$



With the radius A O, the circle may be described on the ground, as in the last Problem.

2. Required the radius of a circle containing half an acre of land.

Ans. $126\frac{1}{2}$ links.

Note. The rules given in the two preceding Problems are the inverse of the rules for finding the areas of the same figures in works on mensuration; and their investigations are given in various works on analytical trigonometry.

Problem IX.

To lay out a given quantity of land in the form of an ellipse.

Rule I.—*When one of the diameters is given.* Divide the given area successively by ·7854 and the given diameter, and the quotient will be the other diameter.

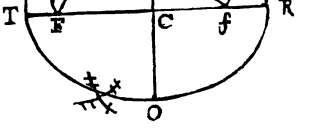
Rule II.—*When the two diameters are required to be in a given proportion or ratio.* Divide the given area successively by ·7854 and the product of the terms of the ratio, and the square root of the quotient, multiplied separately by the terms of the ratio will give the two diameters.

EXAMPLE.

1. Lay out 4 acres in an elliptical form, with a transverse diameter T R of 9 chains.

By Rule I. $\sqrt{\frac{400000}{\cdot 7854 \times 900}} = 566$ { links = C O, the conjugate diameter.

Construction of the ellipse.—Lay off from the centre C of the ellipse, on the transverse T R, the distances CF, C*f*, each equal to the square root of the difference of the squares of the two semi-diameters; take a strong flexible cord, equal to T R, and fix its ends at F, *f*; extend the cord to I, so that it may take the position F I *f*, and keeping the cord continually stretched, with an arrow trace the elliptical curve T I C R O, which will be the required boundary.



In this case $CF = Cf = \sqrt{450^2 - 283^2} = 350$ links.

2. Lay out an ellipse to contain 2*a*. 0*r*. 32*p*., the proportion of the diameters of which shall be as 7 to 4.

By Rule II. $\frac{\sqrt{220000}}{4 \times 7 \times \cdot 7854} = 100$ links; and $100 \times 7 = 700$ links, the transverse diameter, and $100 \times 4 = 400$ links, the conjugate.

Note 1. The investigation of the rules used in this Problem, are given in various works on Conic Sections.

Note 2. The three last Problems are chiefly of use in laying out ornamental grounds, as the inner area of squares in cities and towns, shrubberies, plantations in parks, &c.

Problem X.

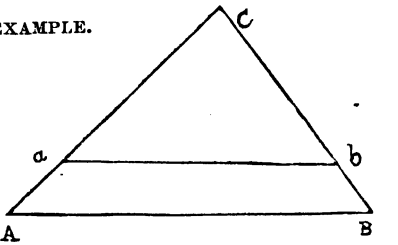
To divide any proposed quantity of land from a triangle, by a line parallel to one of its sides.

Rule.—Since the areas of triangles are as the squares of their like or homologous sides, we shall have

Area $\triangle$ A B C : area $\triangle a b c$ :: A C^2 : a C^2.

See Theor. 9., Chap. I.

EXAMPLE.



1. Let the base A B = 2600 links, A C = 2000, and B C = 1600, cut off 6*a*. 1*r*. 24*p*. by the line *a b* parallel to A B.

From the three sides the area of the triangle A B C is found = 1587450 square links,
and 6*a*. 1*r*. 24*p*. = 640000 square links,
the difference = 947450 = area of $\triangle$ *a b* C.

Then per Rule, 1587450 : 947450 :: 2000^2 : 2387352; and $\sqrt{2387352}$ = 1545 links = *a* C; whence 2000 — 1545 = 455 links = A *a*. Again by similar triangles 2000 : 1600 :: 455 : 364 links = B *b*. Therefore, measure on the ground the distances A *a*, B *b*, just found, respectively from the points A and B; and range the line *a b*, which will divide the given quantity A B *b a* from the triangular field A B C, as required.

Note. In this manner a triangle may be divided into any number of equal or unequal parts, by lines parallel to any one of its sides, by successively subtracting the sums of 2, 3, 4, &c., areas from the area of the triangle, and making the remainders successively the second terms of the proportion.

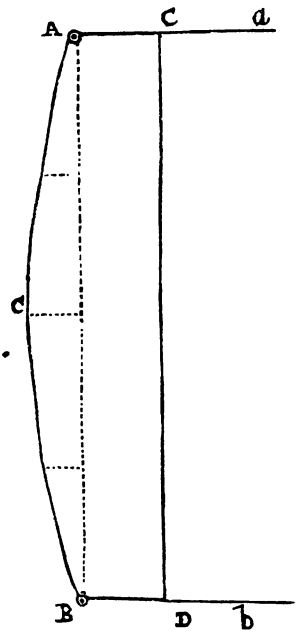
2. The sides of a triangular field are 900, 750 and 600 links; it is required to cut 0*a*. 3*r*. 28*p*. therefrom, by a straight fence parallel to its least side.

Ans. The distance from the ends of the least side, on the largest and intermediate sides, are respectively 211$\frac{1}{3}$ and 176 links.

Problem XI.

To lay off any given quantity of land from an irregular field or common.

Rule.—First find the area of the crooked or irregular part by means of offsets, which area must be deducted from the given area, after which the remaining area must be laid out by means of Problems II, IV and V.



CASE I.—When one side of the field is crooked and two straight and parallel.

EXAMPLE.

Let *a* A *c* B *b* be a portion of a field or common, the side A *c* B being crooked and A *a* B *b* parallel, the length of A B is 2400 links, and the area of the offset piece A *c* B is found to be 227500 square links; it is required to cut off 12 acres from the field by a line parallel to A B.

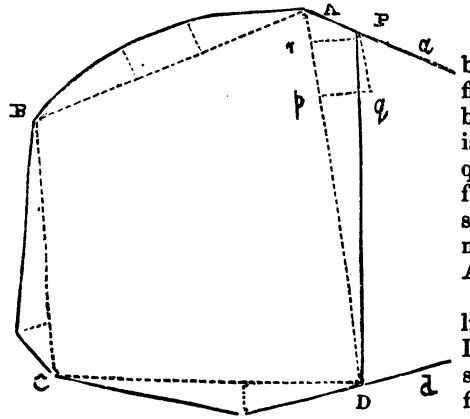
12 acres = 1200000 sq. links
offset piece = 227500 ditto

$$24{\cdot}00 \begin{cases} 2)9725{\cdot}00 \\ 12)4862{\cdot}5 \end{cases}$$

405·2 = AC = BD

Therefore, measure the distances A C = B D = 405½ links on the ground, and range the straight line C D, then shall the space A *c* B D C contain 12 acres.

CASE II.—When all the sides of the field or common are crooked.



EXAMPLE.

Let *a* A B C D *d* be a portion of a field or common, the boundary of which is crooked, it is required to part therefrom 6 acres, by a straight fence running from D towards A.

Measure the four lines AB, BC, CD, D A, taking the offsets on the three first, and finding the

area of the trapezium A B C D in the usual way, which united areas are found to be 570000 square links, the line A D being 810 links.

Hence 6 acres = 600000 square links
Area A B C D and offsets = 570000 ditto

$$81{\cdot}0 \left\{ \begin{array}{l} 9)3000{\cdot}0 \\ 9)333{\cdot}3 \end{array} \right.$$

37·037
2

74·074 links = perpendicular P r

Therefore, apply the perpendicular P r = 74·074 links as in Prob. IV., and range the straight line A P; then shall the 6 acres be parted from the field or common as required.

Problem XII.

To divide from a field or common, bounded by straight fences, any given quantity of land, by a line parallel to one of the straight fences.

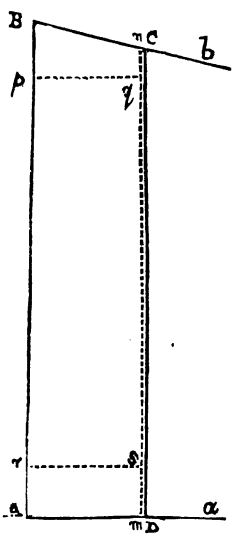
Rule.—Measure the length of the straight fence or line to which the division line is required to be parallel; divide the given quantity by that length, and set the quotient perpendicular to the given straight fence, at two points near the ends thereof, ranging a line through the extremities of the two perpendiculars, and measuring the length thereof; find the area of the trapezoid thus obtained, and take the difference between this area and the given area, which difference, being divided by the last measured line, will give the breadth to be set out perpendicularly from the last measured line, either inwardly or outwardly, accordingly as the difference is in excess or defect of the given quantity.

Example.

Let a A B b be a portion of a field or common with straight fences, it is required to lay out 10a. 2r. 16p. by a line parallel to A B, the length of which is 2200 links.

$$2200 \left\{ \begin{array}{l} 2)1060000 = 10a.\ 2r.\ 16p. \\ 11)5300 \end{array} \right.$$

482 nearly = $p\ q = r\ s$.



Now the perpendiculars $p\,q$, $r\,s$ being set off, and the line $m\,n$ measured, is found to be 2025 links; whence $(2200 + 2025) \times \frac{482}{2} = 1018225$ sq. links, and $(1060000 - 1018225) \div 2025 = 20{\cdot}6$ links, the distance to be set perpendicularly, at both ends of $m\,n$, to the right thereof, because the area cut off by $m\,n$ is less than the given quantity, and the straight line C D ranged through the extremities of these perpendiculars will be parallel to A B, and will cut off the required quantity.

NOTE 1. The three last Problems are much used in the inclosure of commons, and other waste lands, and more especially Problem XII. as the laying out inclosures with parallel fences contributes very much to the economy of cultivation, by thus allowing the ridges to run from the ends of each inclosure, parallel to the two parallel fences.

NOTE 2. Those who are accustomed to operations in analytical trigonometry, will find the following formulæ much more expeditious for laying out land by means of parallel fences, as in Prob. XII., especially where a great number of successive inclosures are required to be laid out:—Let A B $= a$, σ = sine ∟A s = sine ∟B, S = sine $(\sigma + s)$, and A = area A B C D, which is required to be cut off; then,

$$\mathrm{BC} = \frac{1}{\mathrm{S}}\left(a\sigma - \sqrt{\frac{\sigma}{s}(a^2 s\sigma - 2\,\mathrm{A\,S})}\right),$$

$$\text{and } \mathrm{AD} = \frac{s}{\sigma}\,\mathrm{BC}.$$

Whence both B C and A D become known without any preliminary measurement, except that of the side A B and the angles A and B: and by adding the next area to be cut off to A, and calling the sum B, and then substituting B for A in the formulæ for B C, the next following distances $B\,b$ and $A\,a$ may be found; and so on for any number of inclosures.—When A and B are supplemental angles, the lines $A\,a$, $B\,b$, will be parallel, which makes the Problem extremely simple; and when the sum of the angles at A and B are greater than two right angles, S becomes negative, and the lines $A\,a$, $B\,b$, will meet on the other side of A B. In this case the formulæ will become

$$\mathrm{BC} = \frac{1}{\mathrm{S}}\left(\sqrt{\frac{\sigma}{s}(a^2 s\sigma + 2\,\mathrm{A\,S})} - a\sigma\right),$$

$$\text{and } \mathrm{AD} = \frac{s}{\sigma}\,\mathrm{BC}.$$

NOTE 3. *Investigation of the preceding formulæ.* As the formulæ given in Note 2, have never been previously given by any author, it will be proper to

give their investigation here.—Let E be the point where A a, B b, would meet, if prolonged; (this point is not shewn in the figure), then $AE = \frac{as}{S}$, $BE = \frac{a\sigma}{S}$, area of the triangle $ABE = \frac{a^2 s\sigma}{2S}$, and the area of the triangle $CDE = \frac{a^2 s\sigma}{2S} - A$. Now because the triangles ABE, CDE, are similar,

area $\triangle ABE$: area $\triangle CDE :: BE^2 : CE^2$, that is,

$$\frac{a^2 s\sigma}{2S} : \frac{a^2 s\sigma}{2S} - A :: \frac{a^2\sigma^2}{S^2} : CE^2, \text{ whence}$$

$$CE = \sqrt{\frac{2\sigma}{Ss}\left(\frac{a^2 s\sigma}{2S} - A\right)}, \text{ and}$$

$$BC = BE - CE = \frac{a\sigma}{S} - \sqrt{\frac{2\sigma}{Ss}\left(\frac{a^2 s\sigma}{2S} - A\right)}, \text{ or}$$

$$BC = \frac{1}{S}\left(a\sigma - \sqrt{\frac{\sigma}{s}(a^2 s\sigma - 2AS)}\right).$$

$$\text{Also } BE : BC :: AE : AD = \frac{AE}{BE} BC = \frac{s}{\sigma} BC.$$

The other modification of the formulæ for BC, when the point E falls on the other side of A B, is derived by making S negative.

Section II.

On dividing lands or commons, of either equal or variable value, among various claimants, in proportion to their several claims.

Where land is the property of joint purchasers, co-heirs, co-partners, &c.; or where a common is to be divided among the adjoining proprietors in proportion to the values of their several estates; it becomes requisite to adopt the methods of division given in the following Problems, in conjunction with the Problems in the preceding Section. The present Section will, therefore, present cases of considerable complexity; but in order that these may be clearly understood, the simplest cases shall be first presented to the student.

Problem I.

To divide a rectangular piece of ground of equal value throughout, either equally or unequally, among any given number of claimants, by fences parallel to one of its sides.

Case I.—If the parts to which the rectangular space is to

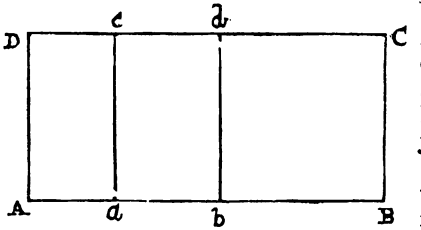
be divided be equal, it will be only required to divide two of the opposite sides of the rectangle into the given number of equal parts, and range the lines for the several fences to the consecutive points of division, and the land will then be divided as required. This appears to be too simple a case to require a diagram.

CASE II.—If the rectangular space is to be divided among several joint purchasers, who have paid unequal sums for the purchase thereof: then use the following

RULE.—As the sum of all the sums paid, is to the length of the side of the rectangle from which the division lines abut, so is any one person's sum to the breadth on the side of the rectangle due to that person; then divide both this side and the one opposite to it, by laying off the resulting breadths, and range lines to the corresponding points, and the rectangle will be divided by parallel lines, as required.

NOTE. This is obviously a question of single fellowship.

EXAMPLE.



Divide the rectangle A B C D, the length of which is 1470 links and its breadth 684 links, among three joint-purchasers A, B and C, who paid for the purchase thereof respectively £120, £150, £220.

£		links		£		links	
120							
150							
220							
494	:	1470	: :	120	:	360	= A a = A's breadth
—	:	—	: :	150	:	450	= a b = B's ditto
—	:	—	: :	220	:	660	= b D = C's ditto

Whole breadth 1470 which proves the work.

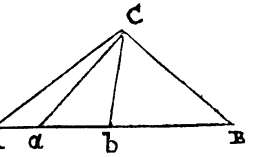
NOTE. In this operation the breadth of the rectangle, which is common to all the three divisions, is not required to be used.

PROBLEM II.

To divide a triangle of equal value throughout, either equally or unequally, among several claimants, who shall all have the use of the same watering-place, situated at one of the angles of the triangular field.

EXAMPLE.

It is required to divide the triangular field A B C among three persons, whose claims therein are as the numbers 2, 3 and 5, so that they may all have the use of a watering-place situate at the angle C; A B being = 1000, A C = 685, and C B 610 links.



The rule in this Problem is the same as in the last.

As 2 + 3 + 5 = 10 : 1000 :: 2 : 200 = A *a*,
— : —— :: 3 : 300 = *a b*,
— : —— :: 5 : 500 = *b* B;

which are the portions of the base A B, belonging to the respective claimants; therefore, if lines be drawn from *a* and *b* to C, the triangular field will be divided in the required proportion, each claimant having the use of the watering-place at C.

NOTE. In solving this Problem it is not necessary to use the lengths of the sides A C, C B, because all the three triangles A C *a*, *a* C *b*, *b* C B have a common perpendicular; and, therefore, their areas are as their bases.

PROBLEM III.

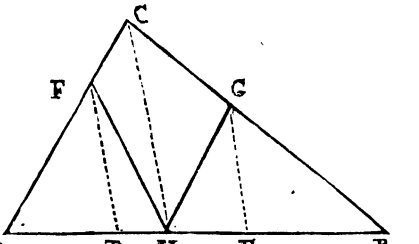
To divide a triangular field of equal value throughout, either equally or unequally, among sundry claimants, by fences running from any given point in one of its sides.

The method of solving this Problem will be best shewn by the following

EXAMPLE.

Divide the triangular field A B C, the sides of which measure 30 chains = A B, 23 = B C, and 19 = A C, equally among three persons, by fences running from an occupation road that meets the side A B at H, which is 14 chains from A, that all the three persons may have the use of the road at H.

Divide AB into three equal parts in the points D, E; from H, (the point where the road meets A B) draw H C; parallel to which draw D F, E G meeting A C, B C respectively, in F and G; and join H F and H G, in which directions fences being made, will divide the triangle as required.



NOTE. This method of solution is founded on the areas of triangles between the same parallels being equal: but it may be solved by actual measurement on the ground, by means of the following preliminary calculation.

ANOTHER METHOD.

As A H : A C :: A D = $\frac{1}{3}$ A B : A F,
that is, 1400 : 1900 :: 1000 : 1357$\frac{1}{7}$ links = AF;
and H B : B C :: E D = $\frac{1}{3}$ A B : B C,
that is, 1600 : 2300 :: 1000 : 1437$\frac{1}{2}$ links = B G.

whence the distances A F and B G may be measured from A and B; and from the points F and G the fences of division may then be drawn to H.

NOTE. When the claims of the several persons are unequal, it will be readily seen that A B is then only required to be divided in the proportion of the several claims, as in the preceding Problem, after which the solution will be the same, as that just given.

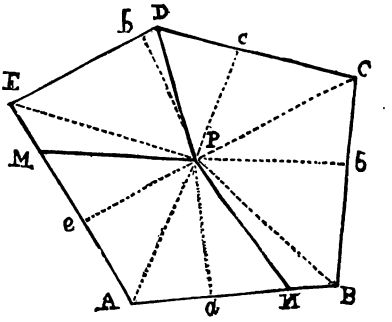
PROBLEM IV.

To divide an irregular field with any number of sides among sundry claimants, so that they may all have the use of a pond, situated at a given point within the field.

NOTE. The method of solving this Problem will be best seen from the method adopted in the following Example, in conjunction with the preceding Problems.

EXAMPLE.

Three persons Q, R, and S, bought a five sided field A B C D E, having a pond therein at P; for which they paid respectively £100, £150, and £200; it is required to divide the field into parts, in proportion to each persons claim, and so that each may have the use of the pond P, the quality of the land being equal throughout. The lengths of the sides and the perpendiculars on each of them to the centre of the pond P are as below, thus constituting five triangles.



A B = 864	P a = 560
B C = 827	P b = 608
C D = 806	P c = 480
D E = 682	P d = 544
E A = 990	P e = 540

By multiplying each side by half the perpendicular thereon, the sum of the five products will be the area of the field; thus,

$$\begin{aligned}
864 \times 560 &= 241920 = \text{area of A P B},\\
827 \times 608 &= 251408 = \text{------ B P C},\\
806 \times 480 &= 193440 = \text{------ C P D},\\
682 \times 544 &= 185504 = \text{------ D P E},\\
990 \times 540 &= 267300 = \text{------ E P A}.
\end{aligned}$$

sum = 11·39572 = acres, the area of the field.

The sums paid for the field by Q, R and S, are as the numbers 2, 3 and 4, the sum of which is 9; therefore,

$$\begin{aligned}
9 &: 11{\cdot}39572 :: 2 : 2{\cdot}53238 = \text{Q's share},\\
— &: \text{------} :: 3 : 3{\cdot}79857 = \text{R's do.}\\
— &: \text{------} :: 4 : 5{\cdot}06476 = \text{S's do.}
\end{aligned}$$

Let D P be assumed the divisional fence between Q and S's shares; then the area of the triangle D P E = 1·85504 acres is less than Q's share, therefore

$$\begin{aligned}
2{\cdot}53238 &= \text{Q's share},\\
1{\cdot}85504 &= \text{area of D P E},\\
\hline
{\cdot}67734 &= \text{difference}.
\end{aligned}$$

This difference must be taken from the triangle E P A to complete Q's share; this is best done by dividing the said difference by half the perpendicular P*e* of the triangle E P A, and the quotient will be the distance E M: thus,

$$\tfrac{1}{2}\,\text{P}\,e = 270)\ 67736\ (250{\cdot}85\ \text{links} = \text{E M}.$$

The distance E M, being nearly 251 links, must now be measured from E on E A, which will give the point M, and, a straight fence being set out from M to P, will cut off Q's share.

The remainder of the triangle E P A, viz. 2·67300 — ·67734 = 1·99566 is less than R's share, therefore,

$$\begin{aligned}
3{\cdot}79857 &= \text{R's share},\\
1{\cdot}99566 &= \text{area M P A},\\
\hline
1{\cdot}80291 &= \text{difference}.
\end{aligned}$$

This difference must be taken from the triangle A P B to complete R's share, as before: thus,

$$\tfrac{1}{2}\,\text{P}\,a = 280)\ 1{\cdot}80291\ (643{\cdot}9\ \text{links} = \text{A N}.$$

This distance being measured from A towards B, will give the point N; and, the fence N P being now set out, will divide

the field as required; the triangles N P B, B P C, C P D making up the exact quantity required for S's share, as may be readily shewn by adding their three areas together, which will prove the accuracy of the work: thus,

	864	= A B
	643·9	= A N
difference	220·1	= N B
	280	= $\frac{1}{2}$ Pa
	61628·0	= area N P B
	251408	= —— P B C
	193440	= —— C P D
sum	5·06476	= S's share, which proves the work.

NOTE 1. If some or all of the fences of the field A B C D E had been crooked, the operation of division would have been the same, excepting that the quantities of the offsets would have to be taken into the account, thus making a little additional calculation. It will at once be seen that this method of division may be extended to any number of claimants, whatever be the shape of the ground to be divided, the dotted lines from P to the angles of the field not requiring to be measured.

NOTE 2. All the surveyors, who have written on the subject of this Problem, use what they call "*guess lines*" to effect the division, and correct the resulting errors by dividing the double quantities in excess or defect by these "guess lines" for the perpendiculars to determine the correct positions of the divisional fences. The student will at once perceive that this is a circuitous, blundering, and unscientific method of proceeding, and should be avoided in every case, except where the boundary is very crooked, and the divisional fences are not required to be continuous straight lines.

NOTE 3. In this Problem the division is effected and proved without the aid of a plan, but it would, perhaps, be better for the satisfaction of the claimants, as well as for the surveyor himself to plan the whole of the work; especially as an error in the work might thus be more readily detected.

PROBLEM V.

To set out from a field or common of variable value, a quantity of land, that shall have a given value, by a straight fence in a given direction.

NOTE. This Problem presents a great variety of cases, the most simple of which shall be first produced.

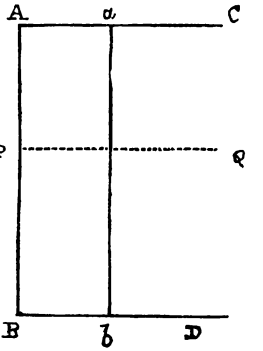
CASE I.—C A B D is a portion of a straight sided field, right angled at A and B, and P Q a right line, also at right angles to A B, dividing the field into portions of different values; it is required to lay off a quantity of land a A B b of given value, by a straight fence a b parallel to A B.

N.B. Lines, such as P Q, are technically called quality lines.

RULE. Multiply A P, P B, respectively, by the value of the adjoining land, take the sum of the products, divide the value of the whole portion to be laid off with five cyphers annexed by this sum, and the quotient will be the required breadth A *a*, or B *b*.

EXAMPLE.

Let A P = 500, P B = 700 links; the value of the land between A C and P Q £50, and that between P Q and B D £80 per acre; it is required to lay out land to the value of £300, by a straight fence *a b* parallel to A B.



Here $500 \times 50 = 25000$
$700 \times 80 = 56000$

81000 = sum of products; and

$$81{\cdot}000 \left\{ \begin{array}{l} 9)30000{\cdot}000 = £300 \text{ with 5 cyphers annexed,} \\ 9)3333{\cdot}3' \\ 370{\cdot}37 = 370\tfrac{1}{3} \text{ links nearly} = A\,a = B\,b. \end{array} \right.$$

NOTE 1. If the "quality line" P Q be not perpendicular to A B, it may be made so by "giving and taking;" especially as the required breadth A *a* may be nearly known by a rough calculation: besides, this liberty with the quality line is usually admitted, since the exact boundary of different qualities of lands cannot be accurately defined.

NOTE 2. This Problem is of great importance, as land of variable quality is very frequently required to be laid out, in the inclosure of extensive commons, by straight fences in given directions for the purpose either of drainage or of irrigation. If any crooked portion of the land to be inclosed, lie to the left of A B, it must be measured, valued, and deducted from the whole value of the land to be laid out; then the Rule, here given, may be applied to the remaining value.

NOTE 3. If the three parallel straight lines A *a*, B *b*, P Q, are not at the same time perpendicular to A B, the Rule just given, will equally apply, excepting that the distance of *a b* from A B must be laid off perpendicularly to A B.

NOTE 4. This rule for laying out land of variable quality has not, to my knowledge, been given by any previous author, the method of performing so simple an operation having been invariably by approximating to the true quantity by means of "guess lines."——*Investigation of the Rule.* Put A P = *a*, P B = *b*, A *a* = B *b* = *x*, *m* = value of land above P Q, *n* = value of land

below it, V = whole value to be laid out, and l = square links in one acre: then $\frac{amx}{l}$ = value of land adjoining A P and $\frac{bnx}{l}$ = value adjoining P B;

$$\text{therefore } \frac{amx}{l} + \frac{bnx}{l} = V; \text{ whence}$$

$$x = \frac{lV}{am + bn}, \text{ Q. E. I.}$$

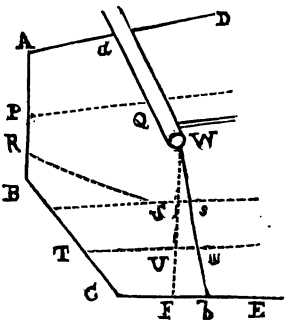
This Rule may be obviously extended to any number of different qualities of land, bounded by parallel quality lines, by multiplying the breadth of each quality by its price, and taking the sum of all the products for a divisor.

CASE II.—When the divisional line is required to pass through a given point W, where an occupation road and watering place is situated, which is to be used by the owners of three or more shares of a common of variable value.

The method of proceeding in this case will be best understood from the following

EXAMPLE.

1. Let D A B C E be a portion of a common of variable value, W a watering place, and a W a road, forming part of the divisional line: it is required to set out a quantity of land of a given value V by a straight fence W b; m being = value of land between P Q and R s, n = value between R s and T u, o = value between T u and C E, and p = value between P Q and A a, the quantity of which is given by boundaries already determined.



First draw W F perpendicular to C E, and find the value of the land to the left of a W F, let this value = v which in this case is assumed to be less than the given value V; therefore, the quantity of land required to the right of W F will be V—v, which put = V′, and having adjusted the portions of the quality lines S s, U u perpendicular to W F, as per Note 1., Case I., let W S = a, S U = b, U F = c, their sum, or the whole distance, W F = s, and F b = x, the symbols denoting the different values of the land being given in the example, and l = square links in an acre: then

$$x = \frac{2\,l\,s\,V'}{a^2\,m + (2\,a + b)\,b\,n + (2\,a + 2\,b + c)\,c\,o} = F\,b;$$

which distance being measured, the straight fence W b may now be set out, and the other shares to the right of a W b may be next proceeded with, according to the method given in this or the following cases.

THE ABOVE EXAMPLE IN NUMBERS.

W S $= a = 460$, S U $= b = 400$, U F $= c = 400$ links; the values are $m = £30$, $n = £40$, and $o = £60$ per acre; and the value V′ $= £80$: required the distance F b or x by the preceding formula.

Here $s = a + b + c = 460 + 400 + 420 = 1280$ links, whence

$$x = \frac{2 \times 1280 \times 100000 \times 80}{460^2 \times 30 + (920+400) \times 400 \times 40 + (920+800+420) \times 420 \times 60}$$

$$= \frac{20480000{\cdot}000}{76635{\cdot}000} = 267\tfrac{1}{8} \text{ links} = \text{F } b,$$

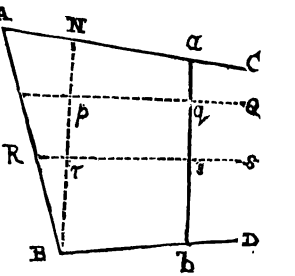
which determines the position of the fence W b.

NOTE. This method possesses the advantage of being practicable on the ground without the help of either a map or guess-lines; however, in the case of the division of commons a map is always necessary for the satisfaction of the several parties interested therein.—*The investigation of formula* here given, is founded on similar triangles combined with the same principle as that in Case I. If there be less than three different qualities of land to be laid out, the symbols referring to the additional qualities, must be made to vanish in the formula; and, if there be more than three qualities to be laid out, the law for the extension of the formula is obvious.

CASE III.—To set out from a field or common of any form, and of variable value, a quantity of land of given value by a straight fence in a given direction.

EXAMPLE.

Let C A B D be a portion of a common, of variable value, from which it is required to set out a quantity of land of a given value V, by a straight fence ab parallel to B N, the quality lines P Q, R S being so adjusted as to be perpendicular to B N at p and r respectively, as per Note 1, Case I., and the value of the land between A C, P Q being $= m$, between P Q, R S $= n$, and between R S, B D $= o$.



First find the value of the land in the triangle A N B, and, if the fence A B had been crooked, the offsets would have to be included; the value of the land to the left of B N being

supposed to be less than is required to make up the given value V. Let the value of the land in the triangle A N B $= v$, then the value of the land still remaining to be set out will be V—v, which put $= V'$, let N $p = a$, $p\,r = b$, r B $= c$, $p\,q = r\,s = x$, cot. of the angle C N B $= 2\,\alpha$, and cot. of the angle D B N $= 2\,\beta$, the symbols denoting the different values of the land being already given in the example, and l = square links in an acre; then

$$x = \frac{2\,l\,V'}{am + bn + co + \sqrt{(am + bn + co)^2 - 4(a + \beta)\,l\,V'}} = p\,q \text{ or } r\,s;$$

which distance may be set out at any two points perpendicular to N B; and, if the land in question be a common, requiring the shares of several claimants to be set out, the quality lines may be again adjusted to the right of $a\,b$, and the next share may then be set out as before.

NOTE 1. The investigation of the general formulæ, just given, is the same in principle as those in the preceeding cases, being only a little more complex, on account of the land to be set out not being rectangular, and therefore involving the solution of a quadratic equation, thus giving an *apparent* complexity to the result which will doubtless startle the generality of surveyors, who are accustomed to use "*guess lines*" when even the most trifling difficulty occurs, thus making repeated approximations to obtain the position of the correct divisional fences, which may be obtained at once by these methods, with even less calculation than by guess lines, without naming the repeated measurements accompanying these bungling operations. It would, therefore, appear that neither the surveyors, nor the writers on this department of surveying, have had the least acquaintance with the Mathematics, otherwise the results here given, which are comparatively void of extreme complexity, would have been known long ago.

NOTE 2. When one or both of the angles C N B, D B N are obtuse, their $\frac{1}{2}$ cotangents α and β will be one or both negative respectively; and when these angles are right ones, their cotangents vanish; and the formula for the perpendicular breadth $p\,q$ or $r\,s$ becomes

$$x = \frac{l\,V'}{a\,m + b\,n + c\,o} = p\,q = r\,s.$$

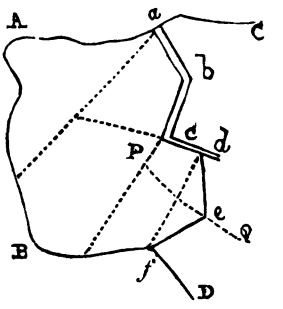
which is the same as the formula given in Case I., as it obviously ought to be.

CASE IV.—When the boundary of the land to be laid out is very irregular, also when part of the divisional fence is predetermined, either for the purpose of drainage or irrigation, and when the quality lines run irregularly.

EXAMPLE.

Let C A B D be the boundary of a portion of common, from which it is required to lay out a portion of land of variable value, part of the divisional fence of which it is desirable to have in the direction of a water course $a\,b\,c\,d$, and the remainder of the divisional fence to run from d to f.

Join *d f*, and find the value of the land included thereby and the other boundaries in question, and, this value being found less than the required value, the remaining land still required, may be determined by assuming two lines, so as to form a triangle on *d f*, as a base, till the correct position of the remaining part *d e f* of the divisional fence shall be ascertained, it being advisable to let the angle *e*, in the required fence, fall on the quality line P Q, for the purpose of more readily calculating the areas of the two triangles formed thereby, and from thence finding their values.



NOTE. It will here be proper to notice that surveyors who are unaccustomed to use mathematical formula, will more readily and accurately make the divisions of commons, &c., by means of assumed or guess lines, than by the methods I have given in the three first Cases of this Problem. Their not having had any practice beyond finding the areas of triangles, &c., gives them little confidence in more scientific methods, which they therefore view as chimeras, or at best as methods very difficult to be understood. It is to be hoped that the very general diffusion of science at the present day will root out this ignorant prejudice.

PROBLEM VI.

CASE I.—*To divide a common of uniform value among any number of proprietors, in proportion to the values of their respective estates.*

In this case, the quantities and values per acre of each proprietor's estate must be determined, by survey if necessary, and the map, with the quantity of the common, must also be prepared; the value of the common, being uniform, is not required.

TO FIND EACH PROPRIETOR'S SHARE.

RULE.—As the sum of the values of each person's estate : the whole quantity of common to be divided : : the value of each person's estate : to that person's share.

EXAMPLE.

Divide a common of 100 acres among three persons, A, B, and C; A's estate is 120 acres at £4 per acre, B's is 180 acres at £5 per acre, and C's 260 acres at £2 per acre.

£		£							
120	×	4	=	480					
180	×	5	=	900					
260	×	2	=	520					

acres				a.	r.	p.	
1900	: 100 ::	480	:	25	1	2·1	= A's share,
——	: —— ::	900	:	47	1	18.9	= B's do.
——	: —— ::	520	:	27	1	18.9	= C's do.
				99	3	39·9	proof

The quantity of each person's share being now determined, the common may be divided by one or other of the methods already given.

CASE II.—*To divide a common of variable value among any number of claimants, in proportion to the values of their respective estates.*

In this case, the map of the common with the quantity of every different quality of land thereon, must be prepared, as well as the quantities and qualities of the several parts of each claimant's estate, in order that the values of both the common and each person's estate may be properly ascertained; the quality lines on the common being marked out by the valuer or valuers, previous to the survey being made.

When the survey of the common is completed and mapped, with the quality lines laid down thereon, the several qualities must be numbered 1, 2, 3, &c., with the values of the land corresponding to each number, as determined by the valuers, who are usually called "Commissioners." Most surveyors use the letters of the alphabet to denote the different values of the land: thus *a* may represent five shillings, *b* six shillings, &c., per acre per annum. By using the letters in this manner a multiplicity of figures, which may be confounded with other numbers, is avoided.

ROADS, QUARRIES, WATERING PLACES, &c., REQUIRED TO BE SET OUT PREVIOUS TO ENCLOSING COMMONS, WASTES, &c.

Before the lands of a common or waste can be divided and allotted, both public and occupation roads must be set out in the most convenient manner; they should be straight and, as far as practicable, at right angles to one another, as this arrangement not only facilitates the division of the land, but contributes greatly to the economy of cultivation with the plough. All old roads that may be deemed unnecessary may be stopped up and

allotted to the different claimants, or diverted into more convenient directions, at the discretion of the Commissioners.

Portions of the common are now to be set apart for quarries, sand or gravel pits, if such abound in the common. The ground, thus set out, is considered as the common property of the several claimants, for the purpose of building, making roads, &c. Also, if there are any good springs or ponds on the common, they must be left unenclosed, in like manner, for common use; or the water must be conveyed from them by drains or channels to more convenient situations, previous to the enclosure of the common.

The lord of the manor in some places claims $\frac{1}{7}$th of the common, in some $\frac{1}{10}$th, &c. His claim, whatever it be, must be next set out, after its value has been determined from the whole value of the common. The lord of the manor will also be further entitled to his share of common, in proportion to his property, in the same manner as the other proprietors.

Lastly, when the roads, watering places, quarries, sand and gravel pits, and manorial rights have been set out, the remainder of the common must be divided equitably, as it respects quantity, quality, and situation, among the proprietors of lands, tenements, houses, cottages, &c., situated in the parish or township where the inclosure is to be made.

THE METHOD OF DIVIDING AND ALLOTTING THE REMAINING PART OF THE COMMON.

Having found the sum of the values of each proprietor's estate, and the whole value of the remaining part of the common to be divided, proceed to find the value of each proprietor's share by the following

RULE.—As the sum of the values of each proprietor's estate, is to the whole value of the common remaining to be divided, so is the value of each proprietor's estate to his share of the value of the common.

It will be quite unnecessary to give an example in this case, as the laying out of the shares of the several claimants, after their respective values have been found by the above Rule, would only be a repetition of the methods already given, on a large scale; and after the work of laying out the several inclosures on the ground has been completed, the last enclosure or share of the common must be of the same value as that assigned by the rule, or so very near to it that the error is of no importance; otherwise a mistake has been made which must be immediately rectified.

For further information respecting the law for the inclosure of commons, waste lands, &c., the student may consult the following

ACTS OF PARLIAMENTS.

GENERAL ACT.

"An Act for consolidating in one Act, certain provisions usually inserted in Acts of Inclosure; and for facilitating the mode of proving the several facts, usually required on the passing of such Acts." (Act 41 Geo. III., chap. 109. 1801.)

Since the General Act, the following Acts have been passed, relating to inclosures.

1. "An Act to amend the Law respecting the inclosing of open fields, pastures, moors, commons, and waste lands in England." (Act 1 & 2 Geo. IV. chap. 23.; April 19th, 1821.)
2. "An Act for facilitating the inclosure of open and arable fields in England and Wales." (Act 6 & 7 Will. IV. chap. 115.; August 20th, 1836.)
3. "An Act to extend the Powers and Provisions of the several Acts relating to the inclosure of open and arable fields in England and Wales." (Act 3 & 4 Vict. chap. 31.; July 23rd, 1840.)
4. "An Act to facilitate the inclosure and improvement of commons and lands held in common, the exchange of lands, and the division of intermixed lands; to provide remedies for defective or incomplete Executions, and for the non-execution of the Powers of general and local Inclosure Acts; and to provide for the revival of such Powers in certain cases." (Act 8 & 9 Vict. chap. 118.; August 8th, 1845.)
5. "An Act to amend the Act to facilitate the inclosure and improvement of commons." (Act 9 & 10 Vict. chap. 70; August 26th, 1846.)

NOTE.—All the above Acts may be procured through any bookseller in town or country.

END OF PART I.

ENGINEERING SURVEYING.

PART II.
LEVELLING.

CHAPTER I.
DEFINITION OF LEVELLING.

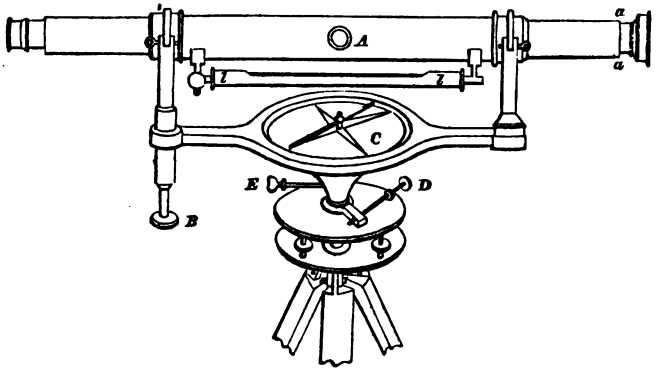
By the art of levelling the inequalities of the upper boundary of any section of the earth's surface may be shewn, and thence may be determined the several heights of any number of points in that boundary, above or below an assumed line, called a level line; though, in reality, this line is a great circle of the earth, and is such as would be derived from a section of the surface of still water.

LEVELLING INSTRUMENTS.

(1.) Levelling instruments all depend on the action of gravity: of these the plumb-line, on which the mason's level depends, is the most simple; but it cannot be used in extensive operations, on account of its practical inconvenience. The fluid, or water level, in all its modifications, is also found inconvenient for extensive practice.

(2.) Spirit levels are now commonly used, as the most accurate instruments for finding the differences of level, or vertical distances between two stations: of these there are three, which we shall proceed to describe, namely, the Y level, Troughton's, and Gravatt's levels.

THE Y LEVEL.



The foregoing figure represents this instrument. A is an achromatic telescope, resting on two supporters, which in shape resemble the letter Y; hence the name of the instrument. The lower ends of these supporters are let perpendicularly in a strong brass bar, which carries a compass box C. This compass is convenient for taking bearings, and has a contrivance for throwing the needle off its centre, when not in use. One of the Y supporters is fitted into a socket, and can be raised or lowered by the screw B.

Beneath the compass box, which is generally of one piece with the bar, is a conical axis passing through the upper of two parallel plates, and terminating in a ball supported by a socket. Immediately above the upper parallel plate is a collar, which can be made to embrace the conical axis tightly by turning the clamping screw E; and a slow horizontal motion can be given to the instrument by means of the tangent screw D. The two parallel plates are connected together by the ball and socket already mentioned, and are set firm by four mill-headed screws, which turn in sockets fixed to the lower plate, while their heads press against the under side of the upper plate, and thus serve the purpose of setting the instrument truly level.

Beneath the lower parallel plate is a female screw, adapted to the staff head, which is connected with brass joints to three mahogany legs, of exactly the same construction as those already described for supporting the theodolite.

The spirit level *l l* is fixed to the telescope by a joint at one end, and a capstain headed screw at the other, to raise or depress it for adjustment.

(3.) Previous to using this instrument the following adjustments must be attended to.

1. *The adjustments of the telescope for parallax and collimation.*

2. *The adjustment of the bubble tube.*

3. *The adjustment of the axis of the telescope perpendicularly to the vertical axis.*

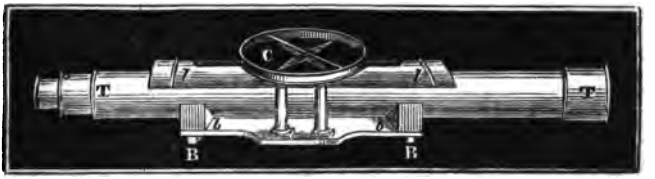
1. *The adjustments for parallax and collimation* have already been described, (p. 76,) being the same as those required for the theodolite.

2. *The adjustment of the bubble tube.*—Move the telescope till it lies in the direction of two of the parallel plate screws, and by giving motion to these screws bring the air-bubble to the centre of its run. Now reverse the telescope carefully in the Ys, that is, change the places of its ends; and should the bubble not settle in the same point of the tube as before, it

shews that the bubble tube is out of adjustment, and requires correcting. The end to which the bubble retires must then be noticed, and the bubble made to return one half the distance by turning the parallel plate screws, and the other half by turning the capstain headed screws at the end of the bubble tube. The telescope must now again be reversed, and the operation repeated, until the bubble settles at the same point of the tube, in the centre of its run, in both positions of the instrument. The adjustment is then perfect, and the clips, that confine the telescope in the Ys should be made fast.

3. *The adjustment of the axis of the telescope perpendicularly to the vertical axis.*—Place the telescope over two of the parallel plate screws, and move them, unscrewing one while screwing up the other, until the bubble of the level settles in the centre of its run; then turn the instrument half round on its vertical axis, so that the contrary ends of the telescope may be over the same two screws, and, if the bubble does not again settle in the same point as before, half the error must be corrected by turning the screw B, and the other half by turning the two parallel plate screws, over which the telescope is placed. Next turn the telescope a quarter round, that it may be over the other two screws, and repeat the same process with these two screws; and when, after a few trials, the bubble maintains the same position in the centre of its run, while the telescope is turned round on the vertical axis, this axis will be truly vertical; and the axis of the telescope being horizontal, by reason of the previous adjustment of the bubble tube, will be perpendicular to the vertical axis, and remain truly horizontal, while the telescope is turned completely round. The adjustment is therefore perfect.

TROUGHTON'S LEVEL.



(4.) In this level the telescope T rests close down upon the horizontal bar *b b*, the spirit level *l l* is permanently fixed to the top of the telescope, and does not therefore admit of adjustment; and the compass box C is supported above the level by four small pillars, attached to the horizontal bar. This construction makes the instrument very firm and compact. Its parallel

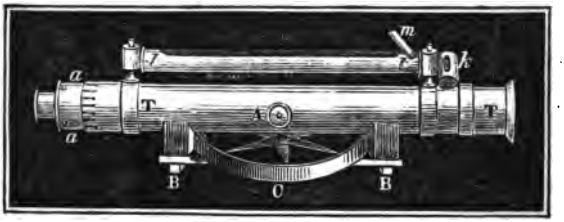
plates and supporting parts are the same as those already described for the theodolite.

The diaphragm is furnished with three threads, two of them vertical, between which the levelling staff may be seen, and the third, horizontal, gives the reading of the staff by its coincidence with one of the graduations marked upon it. Sometimes a pearl micrometer scale is fixed on the diaphragm, instead of the wires. The central division on the scale, in this case, indicates the collimating point, and by the coincidence with a division of the levelling staff give the required reading on the staff; and the scale serves the purpose of measuring distances approximately, and of determining stations nearly equidistant from the instrument, since at such equal distances the staff will subtend the same number of divisions on the micrometer scale.

The line of collimation is set perpendicular to the vertical axis, in the same manner as in the theodolite already described.

This level is preferred by many, on account of its adjustments not being likely, after they are once perfected, to become deranged.

GRAVATT'S LEVEL.



(5.) This instrument is furnished with an object-glass of large aperture, and short focal length; and, sufficient light being thus obtained to admit of a higher magnifying power in the eye-glass, the advantages of a much larger instrument are secured, without the inconvenience of its length:—it is hence called "*the dumpy.*" The diaphragm is carried by the internal tube *a a*, which is nearly equal in length to the external tube. The external tube T T is sprung at its aperture, and gives a steady and even motion to the internal tube *a a*, which is thrust out, and drawn in, to adjust the focus to objects at different distances by means of the milled headed screw A. The spirit level is placed above the telescope, and attached to it by capstain headed screws, one at each end, by means of which the bubble can be brought to the centre of its run, as in the case of

the Y level, already described. The telescope is attached to a horizontal bar in a similar manner as Troughton's level, but room is just left between the telescope and bar for the compass box. A cross level *k* is placed upon the telescope at right angles to the principal level *l l*, by which we are enabled to set up the instrument at once nearly vertical. A mirror *m* mounted upon a hinge-joint is placed at the end of the level *l l*, so that the observer, while reading the staff, can at the same time see that the instrument retains its proper position—a precaution by no means unnecessary in windy weather, or on soft spongy ground.

The telescope is attached to the main bar by capstain headed screws B B, as in Troughton's level, by which the line of collimation is set perpendicular to the vertical axis ; and the instrument is set upon parallel plates, &c., like the theodolite and the levels already described. This level is much preferred and used by many engineers.

(6.) After having treated of the more perfect levelling instruments, it will now be proper to describe the *water level*, a very simple instrument, adapted to give a rapid delineation of any section of the earth's surface, where very strict accuracy is not required. It can be made by any workman, will cost but a few shillings, and requires no adjustment when using it. *It is greatly to be recommended to farmers for determining the levels for draining their lands.*

"*a b* is a hollow tube of brass, about half an inch in diameter, and about three feet long ; *c* and *d* are short pieces of brass tube of larger diameter, into which the long tube is soldered, and are for the purpose of receiving the two small bottles *e* and *f*, the ends of which, after the bottoms have been cut off, by tying a piece of string round them when heated, are fixed in their positions by putty or white lead ; the projecting short axis *g* works in a hollow brass cylinder *h*, which forms the top of a stand : but it may be made in a variety of ways, so as to revolve on any light portable stand. The tube, when required

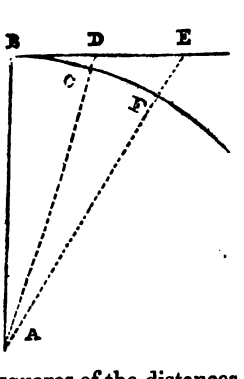
for use, is filled with water, coloured with lake or indigo, till it nearly reaches the necks of the bottles, which are then corked for the convenience of carriage. On setting the stand tolerably level with the eye, these corks are both withdrawn, which must be done carefully, and when the tube is nearly level, otherwise the water will be ejected from one of the bottles; and the surface of the water in the bottle, being necessarily on the same level, gives a horizontal line in whatever direction the tube is turned, by which the vane of a levelling staff may be adjusted."

LEVELLING STAVES.

(6.) The best constructed levelling staff (Gravatt's) consists of three parts sliding one within another, and, when opened out for use, forms a staff 17 feet long, jointed together something after the manner of a fishing rod. The whole length is divided into hundredths of a foot, alternately coloured black and white, and occupying half the breadth of the staff; but for distinctness the lines denoting tenths of feet are continued the whole breadth, every half foot or five tenths being distinguished by a conspicuous black dot on each side, the whole feet being numbered with the figures 1, 2, 3, &c.

CORRECTION FOR CURVATURE.

(7.) Let B D E be a horizontal line, that is, such as would be given by the line of sight of a level, properly adjusted; B C F an arc of a great circle of the earth, and A its centre.



It will at once appear from the figure, that the heights D C, E F, of the apparent level B E, above the true level increase successively from the point B. The height E F of the apparent level above the true, is equal to the square of the distance B E divided by twice the earth's radius A B, that is $E F = \frac{B E^2}{2 A B}$,* and similarly $D C = \frac{B D^2}{2 A B}$, &c., therefore the corrections for curvature, D C, E F, &c., vary as the squares of the distances B D, B E, &c., since 2 A B is a constant quantity.

Taking the earth's radius to be 3979 miles, and assuming the

* The demonstration of this formula is given in my edition of Nesbit's Surveying, p. 348.

distance B D to be 1 mile, then the correction for curvature $D C = B D^2 \div 2 A B = 1^2 \div 7958 = \frac{1}{7958}$ of a mile = 7·962 inches = nearly 8 inches. If the distance B E = 3 miles, then the correction $E F = B E^2 \div 2 A B = \frac{9}{7958} = 71{\cdot}658$ inches, or nearly 6 feet.

Let any distance B D = d in miles, and the correction for curvature for 1 mile be taken = 8 inches = $\frac{2}{3}$ of a foot, which it is very nearly; then

$$\text{correction} = \frac{2 d^2}{3} \text{ feet,}$$

for any distance d in miles:

and let $\frac{c}{80} = d$, c being chains; then

$$\text{correction} = \frac{2 d^2}{3} = \frac{2 \times 12 c^2}{3 \times 80^2} = \frac{c^2}{800} \text{ inches,}$$

for any distance c in chains.

CORRECTION FOR REFRACTION.

(8) The effect of the earth's curvature is modified by another cause, arising from optical deception, namely, refraction; the correction for which varies with the state of the atmosphere, but it may generally be taken at $\frac{1}{7}$ of the correction for curvature, as an average; and since refraction makes objects appear higher than they really are, the correction for it must be deducted from that for curvature.

EXAMPLES.

1. Required the correction for curvature and refraction, when the distance of the object is $2\frac{1}{2}$ miles.

$\frac{2}{3} \times (2{\cdot}5^2) = \frac{2 \times 6{\cdot}25}{3} = 4{\cdot}166$ cor. for curvature.

$\frac{1}{7}$ of which is..... ·693 cor. for refraction.

Difference..... 3·473 feet, cor. required.

2. Required the correction, as in the last example, when the distance is 60 chains.

$60^2 \div 800 = 4{\cdot}5$ cor. for curvature.

$\frac{1}{7}$ of which is... ·643 cor. for refraction.

Difference..... 3·857 inches, cor. required.

3. From a point in the Folkstone road, the top of the keep of Dover Castle was observed to coincide with the horizontal wire of a levelling telescope, when adjusted for observation, and therefore was apparently on the same level; the distance of the instrument from the castle was $4\frac{1}{2}$ miles, required the correction

for curvature and refraction, that is, the true height of the keep of the castle above the point of observation.

$$\frac{2}{3} \times (4{\cdot}5)^2 = \frac{40{\cdot}5}{3} = 13{\cdot}5 \text{ feet, cor. for curvature.}$$

$\frac{1}{7}$ of which ..... = 1·93 feet, cor. for refraction.

Difference ..... = 11·57 feet, cor. required.

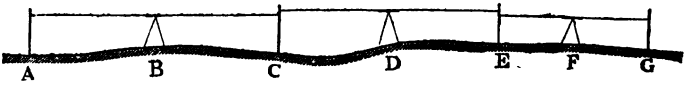
See also the tables for these corrections at the end of the book.

PRINCIPLES AND PRACTICE OF LEVELLING.

To find the differences of the levels of several points on the surface of the earth.

(8.) Before entering on this subject, it will be proper to state that the corrections for curvature and refraction, already explained, are seldom applied in the practice of levelling, the spirit level being usually placed midway between the stations, the levels of which are to be observed, hence the resulting corrections for each station are equal, and therefore the difference of the levels at the two stations is as truly shewn by the difference of the readings of the two staves, fixed thereon, as if the corrections had been made. Thus the trouble of making these corrections is avoided by *simply placing the instrument midway between the two staves.*

(9.) Let it be required to find the difference of level between the points A and G.—A levelling staff is erected at A, the instrument is set up and adjusted at B, another staff is also erected at C, at the same distance from B that B is from A, as nearly as can be judged by the eye; the readings of the two staves are



then noted; the horizontal lines, connecting the staves with the instrument, represent the visual ray or level line of sight. The instrument is then conveyed to D, and the staff that stood at A is now removed to E, the staff C retaining its former position, only its graduated side turned to the instrument, and from being the fore staff at the last observation, it is now the back staff: the reading of the two staves are again noted, and the instrument removed to F, and the staff C to the point G,

the staff at E retaining its position, now in its turn becomes the back staff, and so on to the end of the work, which may thus be continued to any extent. The difference of the readings of the staves at A and C will shew the difference of level between the points or stations A and C, because the visual line of the instrument is virtually level, and the same is true with respect to every two consecutive stations.

EXAMPLE.

Back sight on staff A ..... 10·66 feet
Fore sight on staff C ..... 11·78

The fall from A to C ..... 1·12 difference.

Because when the front reading is the greater the ground falls and *vice versa*.

Back sight on staff C ..... 13·36
Fore sight on staff E ..... 9·16

The rise from C to E ..... 4·20 difference.
Subtract the fall from A to C... 1·12

The rise from A to E ..... 3·08 difference.

Because the rise from C to E is greater than the fall from A to C, their difference shews the total rise.

Back sight on staff E ..... 7·62
Fore sight on staff G ..... 8·16

The fall from E to G ..... 0·54 difference.

This fall taken from the rise from A to E, that is,

3·08
0·54

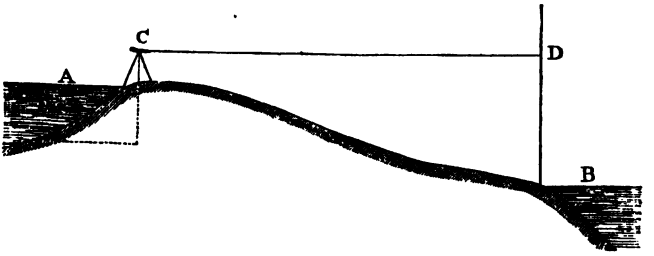
gives the total rise from A to G .....2·54, or nearly 2 feet 6½ inches.

The difference of the sums of the back and fore readings of the staves, will more readily give the difference of level between A and G: thus,

Back sights. feet.	Fore sights. feet.
10·66 at A	11·78 at C
13·36 at C	9·16 at E
7·62 at E	8·16 at G
sums 31·64	29·10
29·10	

2·54 difference of level, the same as before.

(10.) The following is an example of levelling performed at only one operation, and will therefore require the correction for curvature and refraction.



It is required to drain a pond by making a cut to a stream at B, at a distance of 60 chains: let a level be set up and adjusted at C, and then directed to a staff, held upright, at the edge of the water at B. The horizontal line C D represents the line of sight, cutting the staff at D, the reading being 15·84; the height of the instrument above the ground was 4·8 feet, and the depth of the pond 10 feet: what is the difference of level between the bottom of the pond and the surface of the stream?

		ft. dec.
Reading of the staff		15·84
Height of the instrument	4·80	
Depth of pond	10·00	
Correction for curvature and refraction for 60 chains—(see tables at the end of the book)	0·32	
		15·12
Difference of level		0·72

which is little more than 8½ inches, and just a sufficient fall to make the water run freely from the bottom of the pond to the surface of the stream at B; it having been found in practice that a less amount of descent than from 8 to 12 inches per mile produces no efficient current for the purpose of drainage.

TO DRAW A SECTIONAL LINE OF SEVERAL POINTS IN THE EARTH'S SURFACE, THE LEVELS OF WHICH HAVE BEEN TAKEN.

(11.) Let *a*, *b*, *c*, *d*, *e*, *f* and *g* be the several points; then, in order to draw the section to shew the undulations of the ground between *a* and *g*, the distances of the several points from *a*, in

addition to their levels, must be taken; this is usually done during the operation of levelling. These distances, with the back and fore sights, may be arranged in a level book of the following form, which, though not the form practically used, will probably be more clearly understood.

LEVEL BOOK.

Back Sights.	Fore Sights.	Fall.	Rise.	Reduced Levels.	Distances in Chains, and Remarks.
3.50	5·65	2·15		2·15	4·60 at *b* on road.
4·10	10·85	6·75		8·90	7·80 at *c*.
5·04	9·25	4·21		13·11	11·60 at *d*.
3·84	12·91	9·07		22·18	15·20 at *e*.
4·12	7·65	3·53		25·71	bottom of canal, distance 2·16
10·49	3·92		6·57	19·14	21·00 at *f*.
12·96	3·03		9·93	9·21	27·00 at *g*.
44·05	53·26				
	44·05				
diff.	9·21	the same as the last of the reduced levels.			

In this level book it will be seen that the differences 2·15 and 6·75, in the column marked Fall, are added together, making 8·90, thus giving the fall at *c*, in the column marked Reduced Levels: to this sum the succeeding falls are added, one by one, till we get the fall 25·71 at the bottom of the canal, which is the lowest point. Then the differences in the column marked Rise, are subtracted successively from 25·71 for the falls at *f* and *g*; the latter of which is 9·21, the total fall from *a* to *g*, which, agreeing with the difference of the sums of the back and fore sights, shews the truth of the castings. The last column shews the distances of the several points *b*, *c*, &c., from *a*, in chains, with other remarks.

DATUM LINE.

(12.) The section might be plotted by laying off the distances in the last column in the preceding level book on a horizontal line, and setting off their corresponding numbers of feet, in the column marked Reduced Levels, perpendicularly below the line; but it is found inconvenient in practice to plot a section in all cases after this method, as in extensive operations the reduced levels would repeatedly fall above and below the line in ques-

tion, and thus confuse the operation; therefore a line A G called "the datum line" is assumed at 100, 200 feet, &c., below the first station *a*; thus making that line always below the sectional line *a f*, of which a clearer view may be obtained.

(13.) In the following practical level book the rise or fall is respectively added to or subtracted from the assumed distance of the datum line, and the next rise or fall, again added to or subtracted from the sum or difference:—thus 2·15, being a fall, is subtracted from 100 (the assumed distance of the datum line) leaving 97·85 feet, the height of the ground at *b*: the next fall 6·75 is then subtracted from 97·85, leaving 91·10 feet for the height at *c*; and so on to 3·53, which is the last fall:—the next 6·57, being a rise, is added, as well as 9·93;—thus the last reduced level is 90·79 feet, which taken from the datum 100 leave 9.21 feet, agreeing with the differences of the sums of the back and fore sights, and of the sums of the rises and falls, and shewing the work of casting to be correct. Thus are obtained a series of vertical heights to be set off perpendicularly to the datum line, through the upper extremities of which the sectional line must be drawn.

PRACTICAL LEVEL BOOK.

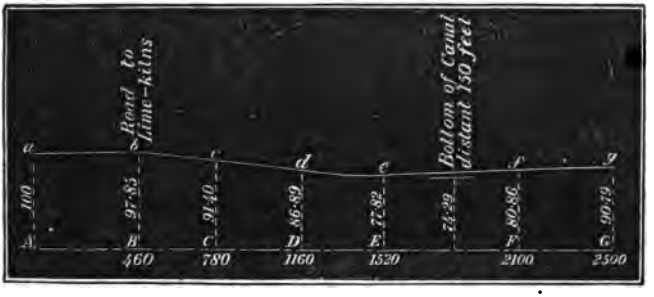
(Datum line 100 feet below the bench mark at A.)

Back Sight.	Fore Sight.	Rise.	Fall.	Reduced Levels.	Distances	Remarks.
feet.	feet.	feet.	feet.	feet.	chains.	
				100·00 D		
3·50	5·65		2·15	97·85	4·60	B M on road to lime kilns.
4·10	10·85		6·75	91·10	.7·80	
5·04	9·25		4·21	86·89	11·60	
3·84	12·91		9·07	77 82	15·20	Bottom of canal, distant 2·80 chains.
4·12	7·65		3·53	74·29	.....	
10·49	3·92	6·57		80·86	21·00	
12·96	3·03	9·93		90·79	27·00	to B. M. at *g*.
44·05	53·26	16·50	25·71	100·00		
	44·05		16·50			
	9·21 diff.	=	9.21	= 9·21		diff. between last reduced level and datum.

In laying down the sectional line from the above columns of reduced levels and distances, the former are always taken from

a much larger scale than the latter, otherwise the undulations on the surface of the ground would, in many cases, be hardly perceptible.

Draw the horizontal line A G, setting off the distances A B, A C, &c., as in the column of distances, that is A B = 4·60 chains, A C = 7·80, &c.; then draw A a = 100 feet, perpendicular to A G and parallel to A a draw B b, C c, &c., setting



off their heights 97·85, 91·10, &c., respectively from the column of reduced levels, and through the points a, b, c, &c., draw the required sectional line a g.

NOTE.—The above operations, though extremely simple, require great care, otherwise, in extensive works of this kind, errors creep in imperceptibly, to check which the agreement of the differences in the level-book is essential.

RUNNING A CHECK LEVEL.

(14.) We shall now give an example of a simple kind to lead the way to more complicated operations. When a section of a line of country has been completed, it is in most cases necessary to check its accuracy by repetition; but in doing this it is seldom requisite to level over precisely the same line of ground, unless there is cause to suspect its general correctness, but to follow the most convenient and nearest route, and at intervals to level to some known points on the exact line of section, which will give their differences of level: the points, thus selected are generally what are called bench marks, being notches cut on gate posts, stumps of trees, mile or boundary stones, or any similarly immoveable objects, contiguous to the line of section, and at frequent intervals. These bench marks are made by the person who first takes the section, and are sometimes previously determined upon. When the section is complete, their relative heights with respect to the base or

datum line of the section becomes known; hence they may be considered as so many fixed points on the line, easily recognisable, from whence any portion of the work may be levelled over and over again.

From what has been said it is clear that in taking running or check levels, the use of the chain and compass attached to the level is not wanted, the distances and bearings having all been previously taken.

An example of this kind of operation is represented in Plate I., where both the ground plan and section are shewn. The strong black line on the plan is that of the section to be checked, and extends from a bench mark at the town A, in a winding direction, to another bench mark in a town B; this originally formed a portion of a more extensive survey. The route taken in proving the work is shewn on the plan by a dotted line, and was confined to the public roads, as being the most convenient route, especially as it crossed the line several times, by which a number of intermediate points could be checked. Previous to giving the particulars of this example, the method of performing the operation of levelling shall be explained more in detail than has yet been done.

The staff-holder must first place his staff perpendicularly on the bench mark from whence the levels are to commence; the surveyor must next set up his spirit level on the most convenient ground that presents itself, and so that he may have an uninterrupted view of the line he intends to level; the station chosen should not be more than four or five chains from the staff-holder, where, having fixed the legs of the level firmly in the ground, the surveyor must adjust his level for observation in the following order:—"first he must draw out the eye-piece of the telescope till he can see the cross wires perfectly well defined; then, directing it to the staff, he must turn the milled headed screw, on the side of the telescope, till he can likewise distinguish, with the utmost possible clearness, the smallest graduations on the staff: that these two adjustments be very carefully and completely performed, is of more consequence than is generally supposed, for on them depends the existence or non-existence of parallax."

"The adjustment of the eye-piece to obtain distinct vision, when once properly made, is not likely to require alteration for the whole day, unless it be accidentally deranged; but that of obtaining distinct vision of the distant staff (together with the one we shall next describe) must be performed at every station as it varies with the distance of the staff.

"Having made the above adjustments perfect, bring the spirit bubble to the centre of its run, which position it must retain in every direction of the telescope; or, in other words, the bubble must indicate a true level during the time the telescope is turned completely round; this is accomplished by bringing the bubble successively over each pair of parallel plate screws, and giving them motion by screwing up one while unscrewing the other to a like extent; but if the telescope is supplied with a cross level, as in that of Gravatt's, the two bubbles being at right angles to each other, they will at once shew which pair of screws require turning, in order to indicate the true level.

"The level being now adjusted for observation, it must be directed to the back staff, and with all possible exactness the foot and decimal fraction of a foot must be noted, with which the central part of the horizontal wire appears to be coincident, which enter in the proper column of the level book. This column should be headed 'Back Sight' as in the preceding example (Art. 13.). As soon as it is registered, see that the bubble has not removed from its central position, and then repeat the observation to ensure that no mistake has been made: this should be invariably done to guard against errors."

The telescope of the level must now be turned round to the fore staff, the object glass and bubble being again adjusted, if required, in the manner already described, and the reading of the staff observed and entered in the level book in the column headed "Fore Sights," verifying the observation as before by seeing that the bubble is still in its place, &c. It may be here proper to remark, that the surveyor's merely walking round the instrument, or accidentally striking one of its legs with his foot, will derange the level indicated by the bubble, especially where the ground is newly tilled or soft and spongy: therefore, the legs of the instrument should always be fixed firmly in the ground. To do away with this inconvenience, as far as possible, Gravatt's level (Art. 5.) has a reflector fixed on the top of the telescope, by which the observer can see both the staff and the reflected image of the bubble at the same time, and thus can make his observation at the instant he sees the bubble in its proper position. The foregoing description of the method of taking levels is general, and applies equally to every kind of levelling operations, we shall hereafter add such additional particulars as will require attending to, when taking levels for the formation of a section. The two first observations being com-

pleted, the surveyor must remove the instrument to the next station and set it up a second time, the person who held the back staff, removing it as far beyond the instrument as the instrument is beyond the staff, which has now become the back staff, and which must not be moved, except that its graduated side must be turned to the instrument. The observations may now be repeated in every respect the same as the two last described, and so on to the end of the work; excepting in cases where "long sights" are taken, in which the correction for curvature and refraction must be applied, as in the example (Art. 10.); but these cases seldom occur in practice. The explanations just given, are of a more detailed character than those given in (Art. 9.), as it must be borne in mind that this work is for the instruction of those who are unacquainted with the subject, and who ought not to be led into tedious details all at once.

The method of taking check levels being now explained, we shall next refer to the example, in which, as already stated, "the levels were taken along the public road shewn by the dotted line (Plate 1,) as being the most convenient route between the town A and B, avoiding the necessity of passing through private property; the strong black line on the plan shews where the original section was taken; the section itself is shewn above the plan, and is drawn from two scales, the one giving horizontal measure, is the same as that of the plan, that is, one inch to the mile; and the vertical scale $\frac{3}{4}$ inch to the 100 feet. From this section it appears that the crown of the bridge at A is 14 feet above the datum line D E of the section, and that the bench mark (a stone by the road side,) at B is 111 feet above the same datum; therefore the difference of level between the two places is $111 - 14 = 97$ feet. Now by referring to our level book, of which we have subjoined a copy, we make the difference of level to be 96·8 feet, differing from the original section only 0·2 of a foot, or 2·4 inches, a quantity that may be disregarded; the inference to be drawn from such a coincidence in the two results is, that the whole of the section between the points in question is sufficiently correct.

LEVEL BOOK FOR RUNNING OR CHECK LEVELS.

Back Sights.	Fore Sights.	Remarks.
0·34	3·16	Back ⊙ on B.M.* on the bridge at A.
5·86	5·61	
4·19	4·24	Forward ⊙ at corner of road leading
5·44	1·20	[to B.
4·96	3·20	
4·73	1·32	At crossing of line.
6·10	2·00	
5·33	3·96	
5·91	1·83	
3·70	0·90	
6·02	1·21	Staff placed on post notched for B. M.
1·21	4·00	At crossing of line.
3·53	6·07	
3·96	5·34	
3·94	4·81	
3·98	6·08	
4·08	4·94	Upon line.
3·90	3·96	
4·84	2·42	
1·54	5·12	
4·69	4·97	
5·04	1·60	
2·24	3·86	Upon line.
7·25	1·89	
4·03	1·30	
9·54	0·19	
6·70	1·70	
9·40	4·06	
6·44	0·38	
140·89	91·32	Sums.
91·32		
49·57		Difference.

* B. M. signifies bench mark.

Back Sights.	Fore Sights.	Remarks.
49·57		Brought forward.
11·00	0·46	
5·98	1·30	
11·12	1·78	
9·84	2·20	
0·18	0·32	Upon line.
4·72	0·10	
8·89	0·77	
10·02	0·92	
10·00	1·03	
8·58	1·19	
9·53	1·18	
9·90	1·68	
9·04	0·35	
10·00	8·52	
3·00	11·55	
3·68	0·88	
7·21	8·75	
1·99	10·48	
0·65	10·00	
4·48	10·44	
7·47	10·30	
1·55	11·70	
2·45	9·88	
3·78	1·04	
6·64	2·65	Forward ⊙ on B. M. called B.
205·27	108·47	Sums.
108·47		
96·80	Diff. =	Difference of levels between A and B.

The difference of the sums of the back and fore sights being nearly 97 feet, proves that the bench mark at B is higher by that distance than the bench mark at A, as shewn by the section.

LEVELS FOR THE FORMATION OF A SECTION.

(14.) The next most simple case that can occur, is to take the levels of a line of country, where the ground plan is already made, and the line of section determined upon and marked out on the plan. Here, in addition to what is required in running or check levels, the distances to the several stations of the levelling staves from the starting point must be measured. Plate II. represents an example of this kind of work, the survey of the land having been completed, and the plan of the fields, &c., drawn: the strong black line A B was the direction determined upon, as the most suitable for an intended line of railroad, and the section was accordingly taken:—bench marks had been previously agreed upon at the extremities A and B, from whence other surveyors could take up the levels, and carry them onwards in both directions.

Two additional assistants are required in this case, to measure the distances of the stave stations along the lines while the operation of levelling goes on, which is the same in every respect as that already described for check levels, excepting that, in this case, the operation is conducted upon the strong black line A B, on the surface plan, a copy of which must be in the surveyor's possession to direct him, and the distances of the several stave stations must be noted in the level book, in the column marked "Distances."

The following is the level book of the example given in Plate II., shewing the manner of keeping it, and also the method of reducing the levels to obtain the actual heights of each station above the datum line E F, which is placed 100 feet below the starting point A, for convenience of drawing the section. The whole operation being similar to that already given at Art. (13), excepting that here and in the preceding example, we give the particular manner of performing the several parts of the field work, in order that it may be clearly understood by those who are unacquainted with the subject, as it is presumed that, in a short time, railways will become the common means of transit, both for passengers and goods, throughout every country of the civilized world.

THE LEVEL BOOK FOR PLOTTING THE SECTION.

(Datum 100 feet below the station A.)

Back Sights.	Fore Sights.	Rise	Fall.	Reduced Levels.	Distances.	Remarks.
feet.	feet.	feet.	feet.	feet.	links.	
				100·00 D		
13·71	7·88	5·83		105·83	519	B. M. side of road.
9·40	16·30		6·90	98·93	1315	
3·87	11·71		7·84	91·09	1542	
2·63	12·41		9·78	81·31	1850	
14·62	0·95	13·67		94·98	2358	
17·00	1·45	15·55		110·53	2698	
10·66	15 40		4·74	105·79	3357	
2·87	17·00		14·13	91·66	3758	
3·40	10·32		6·92	84·74	3976	
5·73	2·24	3·49		88·23	5077	
16·54	0·85	15·69		103·92	5904	
16·08	0·89	15·19		119 11	6124	
14·56	0·73	13·83		132 94	6437	
10·36	14·06		3·70	129·26	7467	
9·84	1·36	8·48		137·72	8369	
9·80	7·00	2·80		140·52	9303	
2·30	10·96		8·66	131·86	——	Centre of road at 215 [links.
10·96	14·46		3 50	128·36	9679	
2·08	15·05		12·97	115·39	9936	
1·75	16·58		14·83	100·56	10164	
1·84	17·10		15·26	85·30	10576	
0·00	7·43		7·43	77·87	11423	Forward ⊙ at corner [of wood.
5·38	3·50	1·88		79·75	13066	
8·50	4·50	4·00		83·75	14954	
5·30	1·36	3·94		87·69	15650	
10·20	9·40	0·80		88·49	17345	
6·86	0·40	6·46		94·95	19135	
11·00	3·96	7·04		101·99	19359	
11·80	3·53	8·27		110·26	19631	
10·53	2·68	7·85		118·11	19841	Forward ⊙ at end of [wood.
8·82	1·38	6·84		124·95	20561	
8·76	2·20	6·56		131·51	21671	
14·00	14·50		0·50	131·01	——	Road at 450 links.
14·50	4·32	10·18		141·19	22710	
9·14	1·00	8·14		149·33	23221	
304·19	254 86	166·49	117·16	100·00		
254·86		117·16				
49·33	—	49·33	—	49·33		Difference between Datum and last Reduced level, or height of B above A.

The several differences of the sums of the back and fore sights, of the sums of the rises and falls, and of the last reduced level and the datum, exactly agreeing, proves the accu-

racy of the arithmetical operation in the preceding level book, all these differences being 49·33 feet, which is the height of B above A, in the section Plate II.

It is advisable for the surveyor to reduce the levels in the field as he proceeds, as it will occupy very little time and can be easily done while the staffman is taking a new position. The surveyor will thus be enabled to detect with the eye if he is committing any glaring error; for instance, inserting a number in the columns of rises, when it ought to be in that of falls, the surface of the ground at once reminding him that he is going downward instead of ascending.

It is seldom the case in practice that the instrument can be placed precisely equi-distant from the back and fore staves, on account of the inequalities of the ground, ponds, &c.; it would appear, therefore, to be necessary, to make our results perfectly correct, to apply to each observation the correction for curvature and refraction as explained in Art. (7): this, we believe, is seldom done, unless in particular cases where the utmost possible accuracy is required, on account of the smallness of such correction, as may be seen by referring to the table at the end of the book, where this correction for 11 chains is shewn to be no more than $\frac{1}{100}$ part of a foot; and as the difference in the distances between the instrument and the fore and back staves can in no case equal that sum, it is evident that such correction may be safely disregarded in practice. Besides it is not necessary to have the level placed directly between the staves while making observations, as it is frequently inconvenient to do so, for reasons just given, nor does a deviation from a line of the staves, in this respect, in the least affect the accuracy of the result.

The distances in the sixth column of the level book are assumed to be horizontal distances, and in measuring them, care should be taken that they are as nearly such as possible, or they must be afterwards reduced thereto, otherwise the section will be longer than it ought to be. For the purpose of assisting the surveyor in making the necessary reduction from the hypothenusal to the horizontal measure, when laying down the section, we annex the following table, shewing the reduction to be made on each chain's length for the following quantities of rise, as shewn by the reading of the staves.

Rise in feet for one chain.	Reduction on one chain in links and decimals.	Rise in feet for one chain.	Reduction on one chain in links and decimals.
1	0·01	11	1·40
2	0·04	12	1·66
3	0·11	13	1·92
4	0·19	14	2·24
5	0·27	15	2·61
6	0·44	16	2·99
7	0·56	17	3·39
8	0·74	18	3·76
9	0·94	19	4·23
10	1·16	20	4·64

TO DRAW THE SECTION.

The levels being reduced, as well as the distances corrected, where required, the surface line may be represented in the form of a section, as shewn in the upper part of Plate II. The vertical and horizontal scales of a section are seldom the same, for the reason assigned in Art. (13.), which produces a caricatured representation of the surface of the section; the vertical scale being so much greater than the horizontal, shews the depths of the cuttings and embankments, required in the execution of railways, canals, &c., with greater clearness than if both scales were alike. The plans and sections of projected work, deposited in the Private Bill Office, to obtain the sanction of the legislature, are mostly drawn to scales of four inches to one mile horizontal and one hundred feet to one inch vertical:—these scales are adopted in the section, Plate II.

To make the section expeditiously, first draw the horizontal line E F, as the datum to which the levels were reduced, take any point E as the starting point, place the feathered edge of the horizontal scale against E F and prick off the several distances in the column headed "Distances," that is, 5·19, 13·15, 15·42 chains, &c., then draw all the perpendiculars by means of a parallel ruler, or by a T square if the paper is properly fixed on a drawing table; and lastly from the vertical scale prick off all the reduced levels or vertical heights corresponding to the several distances, connect the points and the section will be made, after which, write, in vertical lines, the several remarks, as crossing of roads, rivers, &c., that appear in the last column of the level book. The horizontal line A D, called the "datum from bench mark at A," shews the variations of the surface of

the section above and below the point A, from which it was originally plotted by Mr. Simms, his level book being adapted thereto.

CROSS LEVELS.

When a road or river crosses the line of a railway, or canal, cross levels are principally taken, to shew the nature of the surface of the ground, both with respect to improving the main line, if possible, and to shew the approaches of cross roads to the viaducts, when required, and the depth and length of cutting, or the height and length of embankment that will be required, where the main line is either crossed on the level, or over or under by viaducts. The heights of the cross section is usually taken at every 1 or 2 chains length to the distance of 8 or 10 chains on each side of the main line; but if the cross road have a regular slope, it will be sufficient to shew the inclination of the slope, which may be done by a single setting up of the level and staves. The following notes will shew the method of taking a cross section, recollecting always to take the levels from the right to the left of the forward direction of the main line, lest the section, through mistake, should be plotted in the wrong direction.

CROSS LEVELS ON ROAD AT 92·15 CHAINS.
(Section, Plate II.)

Back Sights.	Fore Sights.	Distances.	Remarks.
feet.	feet.	chains.	
1·84	0·16	2·00	
1·02	8·70	3·00	
2·83	7·91	4·00	
5·20	10·63	6·00	
4·13	8·71	7·92	Line crosses reduced level 131·86 feet.
4·04	8·02	10·00	
2·92	7·92	12·00	
3·16	6·04	14·00	
2·17	6·87	15·00	
2·13	7·00	16·80	

The cross section may be plotted on the same scale as the main section; but some engineers adopt a larger one. From what has been already shewn, the student can have no difficulty in plotting from the preceding notes.

PARLIAMENTARY PLAN AND SECTION.

These differ little from what is shewn on Plate II. only in the former the fields, &c., are numbered with an accompanying book of reference to proprietors' names, &c.; and the cross sections are added in the latter, and the gradients put on, as shewn at page 152.

WORKING SECTION.

When the works of a railway, canal, &c., have to be carried into execution, the section must be more minutely taken than would be required in the preceding cases; it is then called a working section. The following are the field notes for such a section. Plate III., *Simms' Levelling*, shews the section of the ground and railway at the extreme end of the line, where the distances terminate at 1103·77 chains, or 13¾ miles and 3·77 chains. The student will be able to plot the section from the following level book and the accompanying directions, without referring to the plate in *Simms' Levelling*. In taking levels for this section, the back and fore sights are not very far distant from each other, and the surveyor will frequently be able to make a number of observations at each setting up of the instrument both in the back and forward directions, the back staff being repeatedly removed nearer the instrument, where the ground varies, and the fore staff also repeatedly removed farther from it: in this manner from seven to ten observations may be obtained at one setting up of the instrument, if required. In the following level book it will be seen it seldom occurred that only one back and one fore sight was obtained at each setting up of the instrument: at the first setting up four forward sights were observed; thus the first back sight was 4·47 and the corresponding fore sights 4·53, 9·22, 5·07, &c. Here the first fall is obtained in the usual manner, that is, by taking the difference between 4·50 and 4·47; the next fall is obtained by taking the difference between the first and second fore sights, that is, between 9·22 and 4·53, this difference is 4·69, which is a fall, because the latter fore sight is the greater; the third difference between the second and third fore sights is a rise, because the latter fore sight in this case, is the less: and so on till we come to the next back sight 6·36, when its corresponding fore sight 1·87 is taken from it, as previously shewn, and the difference placed in the column of rises. The column of reduced levels is obtained as in the preceding examples.

LEVEL BOOK.—WORKING SECTION.

Back Sights.	Fore Sights.	Rise.	Fall.	Reduced Levels.	Distances.	Remarks.
feet.	feet.	feet.	feet.	feet.	chains.	
				270·72		Height above Trinity high water-mark, at London Bridge.
4·47	4·53		0·06	270·66	1033·00	
	9·22		4·69	265·97	1034·00	
	5·07	4·15		270·12	1035·00	
	0·24	4·83		274·95	1036·00	
6·36	1·87	4·49		279·44	1037·00	
6·14	1·47	4·67		284·11	1038·00	Side of falling post of field gate in occupation road.
6·62	2·10	4·52		288·63	1039·00	
	2·24		0·14	288·49	1039·16	
10·42	9·47	0·95		289·44		Lower hinge of gate.
	13·22		3·75	285·69	1039·44	Centre of occupation road
	13·15	0·07		285·76	1039·56	Edge of road.
	8·75	4·40		290·16	1039·66	Top of bank.
	4·48	4·27		294·43	1039·76	Do. do.
	4·32	0·16		294·59	1040·00	
2·44	8·84		6·40	288·19		B.M. south side of line.
	2·83	6·01		294·20	1041·00	
0·74	2·18		1·44	292·76	1042·00	
	5·35		3·17	289·59	1043·00	
6·77	7·28		0·51	289·08	1044·00	
	7·25	0·03		289·11	1044·90	Edge of ditch.
	8·36		1·11	288·00	1044·92	Bottom of ditch.
	3·57	4·79		292·79	1045·00	Stump, top of bank.
3·37	2·75	0·62		293·41	1046·00	
	1·43	1·32		294·73	1047·00	
1·10	2·25		1·15	293·58	1048·00	
	8·88		6·63	286·95	1049·00	Enter alder plantation.
5·65	9·53		3·88	283·07	1049·20	
	11·50		1·97	281·10	1050·00	
5·85	5·52	0·33		281·43	1050·21	
	12·01		6·49	274·94	1051·00	
	12·87		0·86	274·08	1051·48	
	10·77	2·10		276·18	1051·90	Foot of bank, which rises perpendicularly 2 feet.
	8·59	2·18		278·36	1052·00	
	1·40	7·19		285·55	1053·00	
8·22	4·42	3·80		289·35	1054·00	
	2·97	1·45		290·80	1055·00	
	3·39		0·42	290·38	1056·00	
	5·51		2·12	288·26	1057·00	
	7·67		2·16	286·10	1058·00	
5·41	6·68		1·27	284·83	1058·27	Edge of ditch.
	8·56		1·88	282·95	1058·32	Bottom of ditch.
	6·08	2·48		285·43	1058·37	Top of bank.
	12·38		6·30	279·13	1058·54	Foot of bank.
	16·72		4·34	274·79		
2·04	2·66		0·62	274·17	1059·00	
	6·48		3·82	270·35	1059·40	Edge of ditch.

LEVEL BOOK OF WORKING SECTION (*continued*).

Back Sights.	Fore Sights.	Rise.	Fall.	Reduced Levels.	Distances.	Remarks.
feet.	feet.	feet.	feet.	feet.	chains.	
				270·35		Brought forward.
	8·86		2·38	267·97	1059·44	Bottom of ditch.
	6·00	2·86		270·83	1059·52	Top of bank.
	7·58		1·58	269·25	1059·60	Foot of bank.
	10·74		3·16	266·09	1060·00	
3·33	8·24		4·91	261·18	1060·95	Top of bank.
	9·15		0·91	260·27	1061·00	Stump side of bank.
	13·34		4·19	256·08	1061·05	Bottom of ditch.
	11·65	1·69		257·77	1061·10	Edge of ditch.
	12·80		1·15	256·62	1062·00	
3·62	4·13		0·51	256·11	1063·00	
	3·38	0·75		256·86	1063·49	Foot of bank.
	0·50	2·88		259·49	1063·59	Top of bank.
	4·35		3·85	255·89	1063·68	Bottom of drain.
	4·08	0·27		256·16	1063·86	Centre of parish road.
	4·31		0·23	255·93	1064·05	Foot of bank.
	0·57	3·74		259·67	1064·15	Top of bank.
	3·02		2.45	257·22		
0·90	1·40		0·41	256·81	1064·30	Foot of bank.
	2·83		1·43	255·38	1065·00	
	4·41		1·58	253·80	1066·00	
	4·48		0·07	253·73	1067·00	
7·80	7·34	0·46		254·19	1068·00	(Crosses foot path at
	4·41	2·93		257·12	1069·00	[1068·31.)
	0·74	3·67		260·79	1070·00	
10·63	6·43	4·20		264·99	1071·00	
	1·37	5·06		270·05	1072·00	
10·76	4·97	5·79		275·84	1073·00	
	1·12	3·85		279·69	1074·00	
5·42	5·47		0·05	279·64	1075·00	
	4·56	0·91		280·55	1076·00	
	5·00		0·44	280·11	1076·37	Edge of ditch.
	5·56		0·56	279·55	1076·40	Bottom of ditch.
	2·40	3·16		282·71	1076·47	Top of bank.
	2·96		0·56	282·15	1076·54	Foot of bank.
	1·38	1·58		283·73	1077·00	
9·45	0·61	8·84		292·57	1078·00	
8·44	3·33	5·11		297·68	1078·53	Enter plantation.
	0·42	2·91		300·59	1078·57	B. M. on timber stump.
12·78	14·28		1·50	299·09	1078·82	
	10·01	4·27		303·36	1079·00	
	1·26	8·75		312·11	1079·47	
14·49	4·07	10·42		322·53	1080·00	
	2·56	1·21		323·74	1080·08	
	3·35		0·49	323·25	1080·24	
	0·37	2·98		326·23	1080·47	
16·35	1·02	15·33		341·56	1080·98	Top of bank, edge of plan-
	1·32		0·30	341·26	1081·00	[tation.

LEVEL BOOK OF WORKING SECTION (*continued*).

Back Sights.	Fore Sights.	Rise.	Fall.	Reduced Levels.	Distances.	Remarks.
feet.	feet.	feet.	feet.	feet.	chains.	
				341·26		Brought forward.
8.66	5·92	2·74		344·00	1082·00	
	5·14	0·78		344·78	1083·00	
	8·43		3·29	341·49	1084·00	
1·05	4·50		3·45	338·04	1085·00	
	4·94		0·44	337·60	1085·20	
	6·83		1·89	335·71	1085·30	Edge of bank.
	12·54		5·71	330·00	1085·40	Foot of bank.
	16·82		4·28	325·72	1086·00	
1·11	9·04		7·93	317·79	1087·00	
1·18	9·09		7 91	309·88	1088·00	
1·57	9·70		8·13	301·75	1089·00	
1·28	9 58		8·30	293·45	1090·00	
1·44	9·41		7·97	285·48	1091·00	
1·34	9·14		7·80	277·68	1092·00	
1·15	8·12		6·97	270·71	1093·00	
3·04	4·43		1·39	269·32	1093·86	Edge of ditch.
	6·22		1·79	267·53	1093·90	Bottom of ditch.
	5·16	1·06		268·59	1094·00	Stump, top of bank.
	11·10		5·94	262·65	1094·08	Foot of bank.
	9·29	1·81		264·46	1095·00	
8·87	5·77	3·10		267·56	1096·00	At post and rail fence.
4·63	3·46	1·17		268·73	1097·00	Edge of slope.
	7·06		3·60	265·13	1098·00	
1·96	4·60		2·64	262·49	1099·00	Foot of slope.
	4·60			262·49	1100·00	
	3·67	0·93		263·42	1101·49	
4·58	5·03		0·43	262·97	1103·77	Stump, end of curve.
	4·40	0·63		263·60		
	4·58		0·18	263·42		B.M. foot of post.
	1·05	3·53		268·95		Top of said post.

This method of keeping the level book is adopted by many surveyors; the accuracy of the castings is proved by taking the difference of the sum of the back sights and the sum of the last fore sight, leaving out all the intermediate sights, and the difference of the last reduced level, and the same brought forward, and when these differences agree, the castings are correct. Some put the intermediate sights in a separate column, in this case a somewhat different method of casting is adopted:—I have given this method at page 357 of my system of levelling in the ninth edition of Nesbitt's Surveying.

PLOTTING THE WORKING SECTION.

(See Plate III., Simms' Levelling.)

Having drawn the datum line, prick off every chain and number them beneath the datum line to prevent mistakes. Next prick off the distances in the column of distances, and erect the perpendiculars, and just above the datum line, the height at each chain's length should be inserted, from the column of reduced levels; some also insert the heights of the intermediate stations, that the section may be plotted without the fear of errors. The horizontal scale in the example is 1 inch to 5 chains, and the vertical scale 1 inch to 25 feet; these scales are frequently adopted in parliamentary plans. Having laid off all the heights on the perpendiculars, the undulating line forming the surface of the section may be readily drawn, as shewn in the plate referred to, and the description of objects worthy of notice must be added. The section is then prepared for putting on the gradients, &c.

THE METHOD OF LAYING OUT GRADIENTS.

Gradients are evenly ascending or descending portions of a railway; the ascent or descent being always less than 1 foot, reckoned vertically, to 100 feet on the level, or estimated horizontally; thus to afford the means of rapid locomotive traction. Some gradients, however, are level, and such gradients are preferable to any other; but the undulations of the earth's surface prevent their adoption to any great extent, in by far the greater portion of railways.

The extreme left hand point of the section Plate III., being the point of junction with another railway, is the place from whence the gradients are to be laid out: the railway is represented by two parallel lines, the upper one being the surface of the rails, and the lower one the bottom of the ballasting or formation level, being $2\frac{1}{2}$ feet below the surface of the rails. For a short distance the line is level, it then rises at the rate of 20 feet per mile, or one foot in 264, for the twofold object of diminishing the great cutting and getting sufficiently high over the road at stake 1064 to allow headway for public carriages to pass under the railway. From this point the line falls with the same rate of inclination for a considerable distance, the object being to get as low down as convenient further to the eastward where there would be a considerable embankment required; thus reducing the extent of both the cuttings and embankments. Each change of gradient is marked by a strong vertical line from the datum line to the point of change, and

the height written thereon. The quantities of earthwork to form the cuttings and embankments with different slopes should be written on them, as shewn in the example (the method of finding these quantities shall be hereafter shewn); also above the line of figures, denoting the height of the surface above the datum, should be placed the depth of the cutting from the surface to formation level at the same point, or the height of the embankment, as the case may be: these heights and depths are those from which the calculations of the quantities of earthwork are to be made, and therefore must be strictly correct:— these heights and depths are frequently taken by measuring them carefully with the vertical scale of the section, but they may be more carefully obtained by calculation in the following manner. Let it be required to find the depth of the cutting at stake No. 1083, when the height of the surface above the datum is 344·78 feet; at stake No. 1064 the height of formation level above datum is 269·20, from which point the gradient descends at the rate of 20 feet per mile, or 0·25 feet per chain, towards No. 1083; the distance from 1064 to 1083 is 19 chains, which multiplied by 0·25 gives 4·75 for the fall of the railway in the space between the two points; hence the height of the railway above datum at No. 1083 is 269·20 — 4·75 = 264·45; this sum subtracted from the whole height of the surface gives 344·78 — 264·45 = 80·33 for the depth of the cutting at that point, and so on for all the remaining numbers. In the same manner the heights of bridges and viaducts may be found, either below or above the railway, recollecting to subtract the reduced level from the height of the gradient in the former case.

It will here be proper to inform the student that the common, and, perhaps only practical method of laying out the gradients of railways, is by applying one end of an extended silken thread to the section at the commencement of the railway, the other end being so applied that the thread may cut the profile of the earth's surface, so as to leave equal portions of space both above and below the thread, judging by the eye, that the cuttings from the parts above the thread may furnish sufficient materials to fill up the spaces or parts below the thread to form the embankments. If the position of the first gradient, though in in itself favourable, should cause the next gradient to be less favourable, with regard to the extent of cuttings and embankments, the position of the first gradient must be altered to suit the next following gradient, till it be found that the compound result of the cuttings and embankments on the two gradients, or on the successive gradients, as now altered, shall have less of

cuttings and embankments than in the preceding case. In this way it is advisable to change the positions of the gradients till the *minimum* of cuttings and embankments seems evidently to be attained, due regard being had to the limit in the ascent or descent of the gradients, which, as before observed, must not exceed 1 in 100, also keeping in view, at the same time, the proper height for bridges to cross rivers, &c., in the meantime the difficulty of making the excavations being supposed equal throughout the length of the line.

RULE FOR FINDING THE RATE OF INCLINATION OF A GRADIENT.

Divide the horizontal length of the gradient in feet, by the difference of the heights of the gradient at its extremities, above the datum line, and the quotient is the horizontal to a rise or fall of one foot, which is called the rate of inclination of the gradient.

EXAMPLE.

In Plate III.* the second gradient at its commencement, is 261·35 feet, and at its termination it is 269·20 feet above the datum line; thus giving a rise of $269{\cdot}20 - 261{\cdot}35 = 7{\cdot}85$ feet, the horizontal length of the gradient is $1095{\cdot}50 - 1064 = 31{\cdot}50$ chains $= 2079$ feet, whence $2079 \div 7{\cdot}85 = 264$, (fractions being omitted) or a rise of 1 in 264, or of 20 feet per mile, as shewn on the section in the plate referred to.

TUNNELS.

When the excavations reach the depth of 60 feet, and continue at that, or a greater, depth for a considerable distance, the most economical method of proceeding with the work is to make a subterraneous passage called a tunnel, through these deep parts; for it would be next to impossible in many such cases to cut the ground open to the surface. Tunnels, on railways of the narrow gauge, are usually cut to the width and depth of 28 or 30 feet, and on railways of the broad gauge they are proportionably larger. The width and depth of the tunnel are less, if the material to be cut be hard rock. In laying out the gradients the diminished quantity of cuttings, where there are tunnels, must be taken into account. This subject shall be resumed hereafter.

LEVELLING WITH THE THEODOLITE.

The use of the theodolite is sometimes necessary in levelling operations, especially when these operations are required to be

* Simms' Levelling.

conducted over very high and rapidly rising ground, or over steep and almost perpendicular rocks, where the ordinary levelling instrument cannot be fixed. Select a convenient place to fix the theodolite, where the general inclination of the surface of the country changes, without regarding its minor inequalities; then set the instrument level by means of the parallel plate screws, and direct an assistant to go forward with a staff, having a vane or flag fixed to it, of the same height from the ground as the centre of the axis of the telescope of the theodolite. Having gone to the station required, the assistant must hold the vane staff upright, while the observer measures the vertical angle, which an imaginary line, connecting the instrument and staff, makes with the horizon. The instrument and staff should then change places, or, to save time, another staff should take the place of the instrument, the instrument being removed to the former staff, and from thence the angle should again be taken to the second staff; the *mean* of which two angles may be considered the correct angle. This precaution is necessary on account of the variableness of the refraction, and more especially so where the points of observation are at a great distance, and one much higher than the other. The distance on the slope must be measured in the mean time, which, with the mean angle, constitute the hypothenuse and angle at the base of a right angled triangle, in which the base is the horizontal distance between the two stations, and the perpendicular their difference of level, both of which may be readily found by trigonometry, or by laying down the triangle and measuring the parts in question.

In this manner, by considering the surface of every principal undulation as the hypothenuse of a right angled triangle, the operation of levelling may be carried on with great rapidity; but it must be remarked, without pretensions to strict accuracy, —in fact, in this particular the use of the spirit level can never be superseded.

NOTE. The method of finding the difference of levels for railway purposes by the barometer, though frequently recommended, will be found to fail in point of accuracy, on account of the sudden changes in the pressure of the atmosphere, on which depend the indications of this instrument, since 90 feet elevation correspond to only one-tenth of an inch of the mercurial column, which difference has frequently been noticed at the same place, in a very short space of time, the weather at the same time being apparently settled. This method, therefore, can never be relied upon further than as a rough approximation.

CHAPTER II.

THE METHOD OF LAYING OUT RAILWAY CURVES ON THE GROUND.

(1.) ON RAILWAY CURVES IN GENERAL.

In railway practice, the curve adopted is always *an arc of a circle*, and sometimes two, three, or more consecutive arcs of circles of different radii, having a common tangent or tangents at their point or points of junction, as in *the compound curve*. Sometimes the railway curve is composed of two or more circular arcs, having their concavities turned in opposite directions, with a common tangent or tangents at their point or points of junction: a curve, thus composed, is called *the serpentine or S curve*.

NOTE. The arcs of the parabola, ellipse, &c., might with propriety be adopted as the curves of railways, in many cases, and sometimes with advantage; but the method of laying out these curves on the ground is attended with complex calculations; besides a near approach to the arcs of any of these curves may be made by means of the compound curve.

(2.) ON THE DIFFERENT METHODS OF LAYING OUT RAILWAY CURVES.

It has been found in practice, that at least four different methods of laying out railway curves are requisite, partly on account of the degree of scientific skill of, or instruments used by, the engineer or surveyor, and partly on account of obstructions on the ground, as buildings, cliffs, woods, rivers, &c., situated either on the concave or convex side of the curve, or on both sides, or on the curve itself: also on account of pits, swamps, bogs, &c., either wholly or partly preventing the use of the surveying chain.

(3.) ON CURVE RULERS OR MECHANICAL RAILWAY CURVES.

These curve rulers are a series of arcs of circles, usually made of hard wood, as box or mahogany, and sometimes of strong paste-board, with the lengths of their radii in inches marked on them. These curves usually begin with a radius of 2 or $2\frac{1}{2}$ inches, and terminate with one of 160 or 200 inches. The smallest radii commonly increase by half inches up to 10 or 15 inches; then by 1 inch up to 20 or 25 inches; the curves in the middle of the series increase their radii by 2 inches, the next series by 5, and the last by 10 inches. They are used to project curves on railway maps, and to determine the radii of curves previously projected.

The annexed figure is a curve of 15 inches radius. If this curve is used on a map to a scale of six chains to an inch, it will represent a curve of $15 \times 6 = 90$ chains $= 1\frac{1}{8}$ mile radius. When the scale of the map is 2 chains to an inch, it will produce a curve of $15 \times 2 = 30$ chains radius.

In all cases the number of inches in the scale of the map must be multiplied by the radius marked on the curve, to give the radius of the curve on the map.

(4.) THE LIMIT OF THE CURVES OF RAILWAYS.

By an Act of the British Legislature a *minimum* limit has been conditionally appointed for the radii of railway curves. This limitation is rendered necessary, because the centrifugal force, or tendency to motion in a tangental direction, of a railway train of great velocity, in curves of less radius, is so much increased as to hazzard the trains running off the railway on the convex side of the curve. This limitation of the radii of the curves is not required at or near the *termini*, or at or near the principal stations of the railway; since in these situations the speed of the train must be gradually reduced before reaching the curve, for the purpose of stopping. Moreover, curves of less than one mile radius are admitted with safety, by giving a due super-elevation to the rail on the convex side of the curves to counteract the centrifugal force. This, however, is only done, where engineering difficulties require it. See formula for the super-elevation of the exterior rail, Chap. VI.

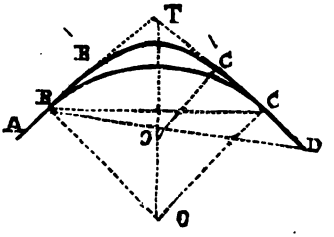
PROBLEM I.

CASE I.—*The positions of two straight, or tangental portions of a railway being given, to determine the radius of the curve that joins them.*

This Problem, at first sight, would seem to admit of an indefinite number of solutions; but when the limitation (see Art. 4.) is considered, in conjunction with the local obstructions of rivers, roads, hills, &c., also of buildings or other valuable property, the problem is frequently limited to one solution, viz., that which presents the fewest of these natural or artificial obstructions to the progress of the curve.

Let A B, C D, be two straight, or tangental portions of a railway, which when prolonged meet at T, and the positions of which are given by a correct map. Apply several of the

curves, described in Art. 3., to touch the lines A B, C D, without cutting them, as at the points B, C, and at the points B′, C′, &c., and let the curves B C, B′ C′, &c., be drawn. Whichsoever of these curves presents the most economical rout for the line of railway ought to be adopted, provided its radius be either equal to, or exceed the minimum limit, viz., 80 chains; unless it be thought advisable to dispense with this limitation, for some of the reasons given in Art. 4. The radius of the required curve, as already stated in Art. 3., is found by multiplying the number on the curve ruler, by the number of chains per inch in the scale of the map.



EXAMPLE.

Let the curve B C by projected by the ruler marked 24, and the scale of the map be 5 chains to 1 inch, required the radius of the curve B C.

$24 \times 5 = 120$ chains $= 1\frac{1}{2}$ miles B O = C O, the required radius.

NOTE. This method of determining the radii of railway curves is usually adopted in practice; but since a circular arc, especially one of large radius, apparently coincides with its tangent for a considerable distance, it is difficult by this method, to find the exact starting and closing points of the curve: besides, it too often happens that the map is not strictly correct; in such cases, the positions of the tangents cannot be said to be given, therefore, this method can only be regarded as giving a rough approximation to the radius and position of the curve.

CASE II.—*The starting point* B *of a railway curve being given, to find its radius geometrically.*

Having prolonged A B, D C, till they meet at T, make T C = C B, and draw B O, C O, respectively perpendicular to A T, D T, meeting in O; then O is the centre, and B O = C O is the radius of the curve, as is evident from the nature of tangents.

Or, bisect the angle A T B by T O; and at B erect B O perpendicular to A T, cutting T O in O; then B O is the radius required.

CASE III.—*When the map is incorrect, to find the radius of the curve by measurement on the ground, the starting point* B *being given.*

Range A B, D C, till they meet at T; measure B T, and take

the angle A T D, one half of which is the angle A T O. Now, in the right angled triangle B T O, B T, and the angle A T O, are given to find B O; which may be readily done by Trigonometry. See *Hann's complete and concise work on this subject.*

CASE IV.—*When the map is incorrect, and the tangents* A B, D C, *cannot be prolonged till they meet, on account of obstructions, the starting point* B *being given.*

From B measure, in the most convenient direction, the line B D, meeting C D in D, the angles T B D, T D B, being respectively taken at the beginning and end of the operation, the sum of which is the supplement of the angle B T D; whence by trigonometry B T may be found, and thence B O, as in the last case.

Or, the radius may be found by the following very concise formula,

$$B\,O = \frac{B\,D \times \text{sine} \angle B\,D\,T^{*}}{\text{ver. sine of sup.} \angle B\,T\,D} = \frac{\frac{1}{2} B\,D \times \text{sine} \angle B\,D\,T}{\text{sine}^2 \text{ of } \frac{1}{2} \text{ sup.} \angle B\,T\,D},$$

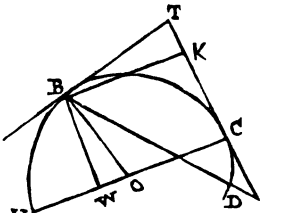
the latter form being adapted to logarithmic computation.

PROBLEM II.

To lay out a railway curve on the ground by the common method.

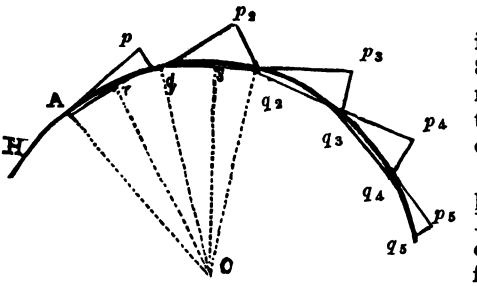
CASE I.—Let H A, q_4 q_5 be the tangental portions of a railway, the extremities A and q_4 of which are required to be united by the circular curve A q_4, to which H A, q_5 q_4 shall be tangents; the radius of the curve being supposed to be found by one or other of the Cases in the preceding Problem, as circumstances may require. See figure on next page.

* *Demonstration.*—Let B T, T C D be tangents to the circle H B C at B, C; O the centre, B O the radius, and C O H the diameter of the circle; and B D a line from the tangental point B to any point D in the other tangent T C D. Draw B K, B W, respectively ⊥ to T D, C H. Then C W = (B K =) B D sine ∠ B D T. But C W is the ver. sine ∠ B O C to radius B O = B O × ver. sine ∠ B O C to radius unity = B O × ver. sine of sup. ∠ B T D to radius unity; ∴ B O × ver. sine of sup. ∠ B T D = B D × sine ∠ B D T,



and $$B\,O = \frac{B\,D \times \text{sine} \angle B\,D\,T}{\text{ver. sine of sup.} \angle B\,T\,D}.$$ Q. E. D.

This formula is due to B. Gompertz, Esq. F.R.S., &c.



Let the radius in this case be 80 chains or one mile; prolong the tangent H A a distance A p = 1 chain; then opposite 80 in table No. 1, at the end of the book, is found 4·95 inches = $p\,q$, which set off at right angles to A p, thus giving the first point in the curve. In the direction A q, measure $q\,p_2 = 1$ chain, and set off $p_2\,q_2$ = twice $p\,q = 4{\cdot}95 \times 2 = 9{\cdot}9$ inches, at right angles to $q\,p_2$; then q_2 is the second point in the curve. This last operation must be repeated till the curve shall have been set out to the point q_4. Lastly $q_4\,p_5$ being measured = 1 chain, in the direction $q_3\,q_4$, the offset $p_5\,q_5$ will be found = 4·95 inches = the first offset $p\,q$, thus proving the accuracy of the work. In this manner the operation is conducted, whatever be the length of the curve.

Case II.—Let A O = r, and δ = A $p = q\,p_2$ = &c., which may be either less or greater than one chain; then the general length of the first and last offsets $p\,q$, $p_5\,q_5$ is $\frac{\delta^2}{2r}$, and the length of each of the other offsets is $\frac{\delta^2}{r}$, or twice the first or last offset; but the length of the offsets given in the table is represented by $\frac{1}{2r}$; therefore, if A p, $q\,p_2$, &c. be taken as 2, 3, 4, &c., chains, the value of $\frac{1}{2r}$ must be multiplied by $2^2 = 4$, $3^2 = 9$, $4^2 = 16$, &c., respectively, to find $p\,q$, and the result, in each case, multiplied by 2 for each of the offsets $p_2\,q_2$, $p_3\,q_3$, &c. In this manner the curve may be set out more speedily, and with less liability to error, on account of the less number and greater length of the lines required in the operation.

EXAMPLE.

Let A O = r = 120 chains, and δ = 4 chains; then $\frac{\delta^2}{2r} = \frac{1}{2r} \times 16 = 3{\cdot}3 \times 16 = 52{\cdot}8$ inches 4 feet 4·8 inches = $p\,q$; whence 4 feet 4·8 inches × 2 = 4·8 feet 9·6 inches = $p_2\,q_2 = p_3\,q_3$ = &c.

Note 1. When the curve has been correctly set out, as in Case II., the inter-

mediate stumps may be put in at the end of every chain, if required, by the method given in Case I. The distances of the intermediate stumps, thus put in, will not, in most cases, exceed a fraction of an inch; because the lengths of the offsets $p\,q$, $p_2\,q_2$, &c., is so small, that the curvilinear lengths $A\,q$, $q\,q_2$, &c. can never greatly exceed those $A\,p$, $q\,p_2$, &c.

NOTE 2. The method given in Case II., is sufficiently accurate when δ does not exceed $\frac{1}{10}$ of the radius of the curve. Besides, at the closing point of the curve, as at q_4, the distance $q_3 p_4$ is most commonly less or greater than δ. Let $q_3 p_4 = d$; then the offset $p_4 q_4$, at the end of the curve is $= \frac{(\delta + d)d}{2r}$; and, when $\delta = 1$ chain, $p_4 q_4 = \frac{(1 + d)d}{2r}$; or the tabular number for the given radius must be multiplied by $(\delta + d)d$, or by $(1 + d)d$, according as $A\,p$, $q\,p_2$, &c., is taken $= \delta$ chains or 1 chain, to give the last offset $p_4 q_4$; $\frac{1}{2}$ of which is $= p_5 q_5$, the offset to the tangent $q_4 q_5$.

EXAMPLE.

Let $r = 120$, and $\delta = 4$ chains, as in the last example, and let $q_3 p_4 = d = 2{\cdot}68$ chains; then $p_4 q_4 = \frac{1}{2r} \times (\delta + d)\, d = 3{\cdot}3 \times (4 + 2{\cdot}68) \times 2{\cdot}68 = 59{\cdot}07$ inches; $\frac{1}{2}$ of which, viz., 29·535 inches is $= p_5 q_5$.

NOTE 3. When δ exceeds $\frac{1}{10}$ of the radius r of the curve, the following formula ought to be used for finding the offsets,

$$p\,q = r - \sqrt{r^2 - \delta^2},$$

$$\text{and } p^2 q^2 = \&c. = \frac{\delta^2}{\sqrt{r^2 - \frac{1}{4}\delta^2}}.$$

See the investigation at the end of these notes.

EXAMPLE.

Let $r = 40$ and $\delta = 4$ chains; then $p\,q = r - \sqrt{r^2 - 8^2} = 40 - \sqrt{40^2 - 4^2} = 40 - 12\sqrt{11} = {\cdot}2005$ chains $= 158{\cdot}8$ inches nearly; which differs from 4^2 multiplied by the tabular value of $\frac{1}{2r} = 158{\cdot}4$ by only ·4 or $\frac{2}{5}$ of an inch, or little more than $\frac{1}{400}$ part of the whole offset. Also $p_2 q_2 = \frac{\delta^2}{\sqrt{r^2 - \frac{1}{4}\delta^2}} = {\cdot}403546$ chain $= 319{\cdot}6$ inches; which differs from $\frac{1}{2r} \times 2 \times 4^2 = 316{\cdot}8$ by 2·8 inches, or only $\frac{1}{114}$ part of the whole offset. In this case, it must be remembered that δ is taken $=$ equal $\frac{1}{10}$ of the radius without material errors; but when it is taken at or near $\frac{1}{15}$ or $\frac{1}{20}$ of the radius, then errors will nearly vanish.

NOTE 4. The offsets to the curves, in the diagrams to this and the following Problems, are drawn much larger than they ought to be, in order that they

may be shewn more clearly; indeed a due proportion, in this particular, would require diagrams of a larger size to shew the several parts distinctly. Besides, only a small part of the operation, for a curve of great length, is shewn; but as the whole of the work, except the first and last offsets, is alike, to shew more would be unnecessary. These circumstances the student will at once perceive; and draw the diagrams to a large scale for practice.

NOTE 5. When the curve has been set out, the rods or arrows that mark the points q, q_2, &c., must be taken out; and strong stumps, from 16 to 18 inches in length and $1\frac{1}{2}$ inch square, must be put in their places, firmly in the ground, to mark the position of the line. The same kind of stumps must also be put at the end of every chain in the straight portions of the line; and the extremities of every curve marked with larger stumps, or two or three of the ordinary stumps put down together.

NOTE 6. By this method the greater part of both British and foreign railway curves have been laid out. It was invented by the author about 27 years ago, when the Stockton and Darlington Railway was laid out, and eagerly adopted by engineers, as it involves very little calculation, and does not require the use of a theodolite. It is, however, defective in practice, on account of its requiring so very many short lines connected together, as errors will unavoidably creep in and multiply, and more especially so where the ground be rough, thus the curve has frequently to be retraced several times before it can be got right. Hence the author prepared the methods in the following Problems. *See Remarks on the Methods of Laying out Railway Curves at the end of these Problems.*

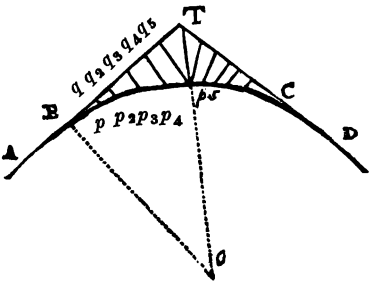
Investigation of the formulæ used in this and the following Problems.—Put $\pi = p\,q$, (see last fig.) the other symbols remaining the same as before; and draw $O\,r$, $O\,s$, respectively perpendicular to $A\,q$, $q\,q_2$; then by similar triangles $AO : A\,r\,(= \frac{1}{2}A\,q) :: A\,q : \frac{A\,q^2}{2\,A\,O} = p\,q = \pi$. But (Euc. I. 47.) $A\,q = \sqrt{\delta^2 + \pi^2}$, $\therefore \pi = \frac{\delta^2 + \pi^2}{2\,r}$, or $\pi^2 - 2\,r\,\pi + \delta^2 = 0$; whence $\pi = r - \sqrt{r^2 - \delta^2}$. When r is large, and δ only 1, 2, or 3 chains, the value of the said expression is very nearly $= r - \frac{\delta}{2\,r}$; whence $\pi = r - \left(r - \frac{\delta}{2\,r}\right) = \frac{\delta}{2\,r}$, which is the formula for the first offset $p\,q$, as practically used.

Now, put $\pi = p_2 q_2 =$ &c., then $q\,q_2 = \sqrt{\delta^2 + \pi^2}$; cos. $\angle p_2 q\,q_2 =$ cos. $\angle p_2 q_2 q_2 = \frac{\delta}{\sqrt{\delta^2 + \pi^2}} =$ cos. $\angle q\,q_2 q_2$; whence $\frac{1}{\sqrt{2}}\left(1 - \frac{\delta}{\sqrt{\delta^2 + \pi^2}}\right)$ = cos. $\angle_1 s\,q\,O =$ sine $\angle s\,O\,q$; and $\frac{\pi^2}{2}\left(1 - \frac{\delta}{\sqrt{\delta^2 + \pi^2}}\right)^2 = \frac{1}{4}(\delta^2 + \pi^2)$. By reducing this equation, rejecting the powers of π above the second, (because π is always in practice very small with respect to δ, and much more so with respect to r,) there results $\pi^2 = \frac{\delta^4(4\,r^2 - \delta^2)}{4\,r^4 - 8\,r^2\delta^2 + 3\,\delta^4} =$ (approximately) $\frac{\delta^4}{\sqrt{r^2 - \frac{7}{4}\delta^2}}$; and when δ is very small compared with r, π^2 is very nearly $= \frac{\delta^4}{r^2}$ or $\pi = \frac{\delta^2}{r}$, which is the formula given for the offsets, $p_2 q_2$, $p_3 q_3$, &c., as commonly used in practice.

Problem III.

To lay out a railway curve on the ground, by offsets from its tangents, no obstructions being supposed to prevent the use of the chain on the convex side of the curve.

Case I.—*When the length of the curve does not exceed* $\frac{1}{4}$* *of its radius.*



Let A B, D C be straight portions of a railway, the points C and B being required to be joined by a circular curve B C, to which A B DC shall be tangents, the radius B O of the curve being supposed to be found by one or other of the cases in Prob. I., as circumstances may require.

Range the tangents A B, D C till they meet at T; and let the radius B O = 80 chains = 1 mile; measure on B T the distance B q = 1 chain; and, at right angles to B T, lay off the offset $q\,p$ = 4·95 inches, by Table No. 1, as in Problem II.; then p is the first point in the curve. Next measure $q\,q_2$ = 1 chain, and lay off the offset $p_2\,q_2 = 4{\cdot}95 \times 4$ for the second point in the curve. The successive offsets, at the end of every chain, being 4, 9, 16, &c., or 2^2, 3^2, 4^2, &c., times the first offset $p\,q$.

When the offsets have been thus laid out, till the last one $q_5\,p_5$ falls little short of T; lay off the same offsets on T C as were laid off in B T, but in an inverted order, making the first distance on T C = T q_5; thus completing the curve to C.

Note. It can rarely happen in practice that the last offsets, from both tangents, will meet at the middle point p_5 of the curve, as shewn in the figure; but will either intersect one another or fall short of the middle point; this is a matter of no consequence, although some engineers require the stumps that mark the line to be placed regularly one chain asunder: the method of doing this shall be shewn hereafter.

* When the curve is longer than $\frac{1}{4}$ of its radius, the offsets in most practical cases become inconveniently long as well as not sufficiently accurate, by the method here given for finding them. See Demonstration to Problem III. p. 14, of *Baker's Railway Engineering.*

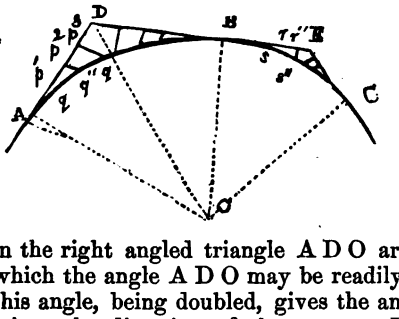
EXAMPLE.

Let the radius of the curve be 160 chains, required the offsets at the end of every chain, from the tangent to the curve.

$p\ q$ (per Table No. 1.) = 2·475 inches.
p_2q_2 = 2·475 × 4 = 9·9 ———
p_3q_3 = 2·475 × 9 = 22·275 ———
p_4q_4 = 2·475 × 16 = 39·6 ———
&c. &c. &c.

CASE II.—*To lay out the curve when it is any required length.*

In a long curve (of which there are some more than two miles in length) the tangents, if prolonged to their point of meeting, would necessarily fall at a great distance from the curve, thus giving an inconvenient length to the offsets which in practice should never exceed two chains. To remedy this inconvenience the curve must be divided into two or more parts, by introducing one or more additional tangents, thus the offsets may be confined within their proper limits. In the annexed figure the curve A C is divided into two unequal parts at D; at which point the tangent D B E is introduced, on the most convenient ground to meet the tangents A D, C E in D and E. The tangent A D must first be measured to an extent not exceeding $\frac{1}{8}$ of the radius A O. Then in the right angled triangle A D O are given A O, D O, from which the angle A D O may be readily found by trigonometry: this angle, being doubled, gives the angle A D B, which determines the direction of the tangent D B E. This done, the offsets to the curve may be laid off, as in the last case, the order of the offsets being inverted on D B and again on D E, &c.



EXAMPLE.

Let A O = 160 chains, and A D = 20; then by trigonometry,—

As 20 (A D) log. ..... 1·30103
: 160 (A O) ..... 2·20412
:: radius..... 10·00000
: tan. $\frac{1}{2}\angle$ A D O = 82° 52$\frac{1}{2}'$ ...10·90309
2

$\angle$ A D B = 165° 45′ whence the direction

of D C E becomes known; the offsets will be the same as in the last example. It would scarcely be necessary either to calculate or take the angle B E C, had it not been advisable to do so for the sake of checking the accuracy of the work.

NOTE. If the length of the curve be considerable, it may be thus divided into 3, 4, or more parts, according to its length.

CASE III.—*When a curve is set out from several tangents, as in the last Case, to make the last and first offsets on any two consecutive tangents meet at one point in the curve.*

This method may be advantageously adopted when the distances of the stumps are required to be equal. Measure on A D any number of chains less than $\frac{1}{8}$ of the radius A O, and find the length of the last or longest offset on A D, which suppose to be $p_3\ q_3$. Then A O : A p_3 :: $p_3\ q_3$: p_3 D; whence the distance p_3 D becomes known, which must be added to A p_3 to give the full length of the tangent A D.

EXAMPLE.

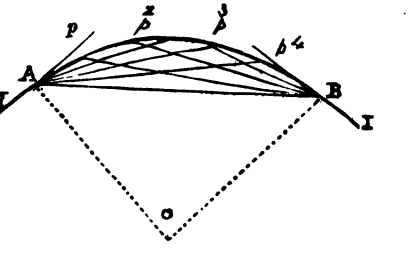
Let A O = 140 chains; then A $p_3 = \frac{1}{8}$ A O = $17\frac{1}{2}$ chains; and by taking the nearest whole number of chains, viz., 17 chains for the length of p_3, we shall have $p_3\ q_3 = \frac{17^2}{2 \times 140} =$ 1·03 chains, and as A p_3 is little less than $\frac{1}{8}$ of A O, p_3 D is sufficiently near to $\frac{1}{8} \times 1{\cdot}03 = 14$ links: whence A D = 17 chains + 14 links = 17·14 chains nearly. The remaining parts of the calculation are the same as in the last Case.

PROBLEM IV.

To lay out the curve, where obstructions, such as water, swamps, pits, &c., prevent the use of the chain, by two theodolites.

CASE I.—*When the points in the curve are any assumed angular distance apart.*

Let A B be the curve; H A, I B tangents at extremities; A O its radius; and p, p_2, p_3, &c., several equidistant points in the curve, from each of which two lines are drawn to the ends A, B of the curve. Let the angle, made by



prolonging the tangents till they meet, be determined, which is the supplement of the angle A O B; and let this angle be divided into any number of equal parts, as $A p$, $p p_2$, $p_2 p_3$, &c., which are each double the measure of the angles $A B p$, $p B p_2$, &c., or double the angles $B A p_4$, $p_4 A p_3$, &c., by the nature of the circle, also the sum of the angles at the base A B of each of the triangles ABp, ABp_2, &c., is equal to half the $\angle$ A O B: hence the following method of setting out the curve:

EXAMPLE.

Let the angle made by the tangents H A, I B when prolonged be 100°; then the angle $A O B = 180° - 100° = 80°$; hence the sum of the angles $B A p$, $A B p = 80° \div 2 = 40°$; therefore, if the angle $A B p$ be taken $= 1°$, the angle $B A p = 40° - 1° = 39°$. These angles may be set out, by having two theodolites fixed at A and B at the same time, and, the corresponding directions $A p$, $B p$, being fixed upon at the same instant by means of signals, may be continued by poles till they intersect at p, which will be the first point in the curve. Secondly, take $\angle A B p_2 = 2°$; then $\angle P A p_2 = 40° - 2° = 38°$; and the lines $A p_2$, $B p_2$, being continued, as before, till they intersect in p_2, will give a second point in the curve. In the same manner a succession of points may be found, by continually increasing the angle at A by 1°, and diminishing that at B by the same quantity, till the former angle becomes 39° and the latter 1°, thus giving 39 points in the curve.

NOTE. It may here be proper to remark, that the obstructions are sometimes so great, that some of the points in the curve cannot be found: in a case of this kind, it will be necessary to defer the finding of such points, till the works of the railway are so far advanced as to present a better opportunity.

CASE II.—*When the points in the curve are at a given distance apart.*

Let r be the radius, and δ be the given distance of the points in the curve $= A p = p p_2 =$ &c.; and let $\angle A B p$ be taken = an arc whose sine is $\frac{\delta}{2r}$* $= \delta \times$ tabular length of the first offsets used in the two last Problems. This angle must be doubled, tripled, &c., to give the successive angles to be taken at B; which angles, being successively subtracted from $\frac{1}{2} \angle A O B$, will give the corresponding angles to be taken at A; which may be set out as in the preceding case.

EXAMPLE.

When $r = 80$, $\delta = 1$ chain, and the angle between the tan-

* For the investigation of this and other formulæ in this work, see the foot notes in *Baker's Railway Engineering.*

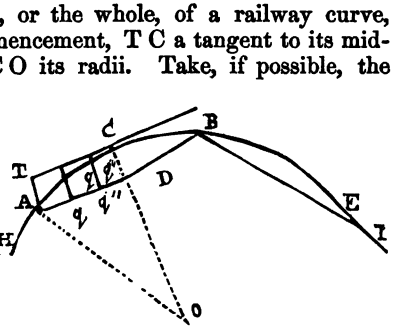
gent prolonged the same as in the last example; then $\angle ABp$ = arc to natural sine $\frac{1}{2r}$ $(= \frac{1}{160} = \cdot00625) = 0° 21' 29''$; whence the other angles are readily found; which, being set out, will give the consecutive points in the curve, each one chain apart, except the last, which suppose to be p_4 B and which may be either 1 chain, or a fractional part of a chain, according to the magnitude of $\angle$ A O B.

NOTE. *Mr. Rankine's method of setting out the curve.*—As this method is evidently a modification of the one just given, it shall be here explained. It is founded on the property here given $\angle ABp = \angle pBp^2 = \&c. = \angle ATp$ (the tangent HA being prolonged to T, which is not shewn in the figure) $= \&c. =$ arc to natural sine $\frac{\delta}{2r}$; in which δ is always taken = 1 chain. A theodolite is fixed at A, the $\angle pAT$ is taken, and the distance $Ap = 1$ chain is set out from A to p, the point p being brought exactly into the direction of the visual axis of the instrument; the $\angle p_2AT$ is next set out $= 2 \times \angle pAT$, and the distance $p_1p_2 = 1$ chain, applied in the same manner as Ap, and so on to the end of the curve. This method, though theoretically accurate, is even more objectionable in practice than my common method, given in Prob. II., Case I., on account of the multiplicity of small angles and short lines required; while errors, committed at the beginning of the work, will gradually increase as it proceeds, whether the errors result from taking the angles or the short distances Ap, pp_2, &c., and especially in the distances, if the ground be rough or uneven. Besides, this method cannot be improved by taking longer chords, as in Prob. II. Case II., without a chain of unweildy length. I would, therefore, not recommend this method, especially to trace a curve of great length, and where the tangent at its termination cannot be changed in position, to accommodate the almost unavoidable errors. Moreover, *Rankine's method would not have been noticed in this work, had it not been put forth by several authors as a perfect one.*

PROBLEM V.

To set out the curve by offsets from its chord or chords, where obstructions, on its convex side, prevent the use of the preceding methods.

Let A C B be a portion, or the whole, of a railway curve, H A a tangent at its commencement, T C a tangent to its middle point C, and A O, C O its radii. Take, if possible, the chord A B an even number of chains; find the successive offsets, corresponding to the radius A O and the tangent T C = A D = $\frac{1}{2}$ A B, the last offset T A will be = C D; from C D subtract the successive



offsets, and the remainders will be the offsets $p_2 q_2$, $p q$, &c., which must be set off, in an inverted order from A to D, and their order must be again inverted in setting them off from D to B. If the curve shall not yet be completed, the operation may be continued by taking other chords, as B E.

NOTE 1. It will be seen that the tangent T C, which is parallel to A B, is not used in the operation, further than to explain the nature of the method of finding the offsets.

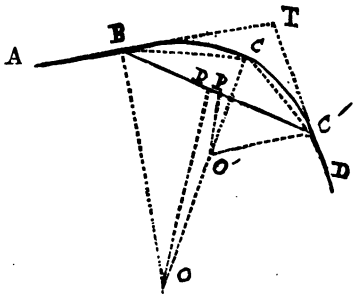
NOTE 2. This method may be advantageously used where a winding river, buildings, cliffs, &c., are close to, or protrude in some places over, the curve; thus excluding the use of any other method, except that in the preceding Problem, which is not always convenient, on account of its requiring two theodolites.

NOTE 3. The curve may be laid out independent of all the preceding methods, where the map can be relied on for accuracy, and where, at the same time, there is a great number of buildings, or objects marked down on the map, by placing marks on the points of the buildings, &c. cut by the projected curve.

PROBLEM VI.

1. *To find the radii of the compound curve by curve rulers.*

The compound curve has been defined in Art. (1.), page 156. Let A B, C′ D be the straight portions of a railway which are to be joined by a curve passing through or near the point C.



Apply several of the curve rulers so as to touch A B at B, without cutting it; and that which passes through, or very near to C, must be selected as a portion of the line of railway. Apply another series of curve rulers till one be found to touch the curve B C at or near C, and the tangent C′ D at C′. Then find the radii B O, C O′ of the two curves as in Prob. I.

2. *When the compound curve has several radii terminated by given points.*

If the nature of the ground is such as to require three, four, or more curves of different radii; also if C′ be required to be a given point; the method of determining the several radii, &c. will be sufficiently obvious from what has been already shewn.

NOTE. Having determined the radii, as B O, C O′, &c., the tangent points, as C′ and the junction points, as C, &c., with the common tangent at the junction points, the different portions of the curve may be set out by the methods given in the preceding Problems, according to circumstances as respects obstructions, &c.

Problem VII.

To determine one of the two radii of a compound curve by calculation, the other radius and the tangent points being given. (See figure to last Problem.)

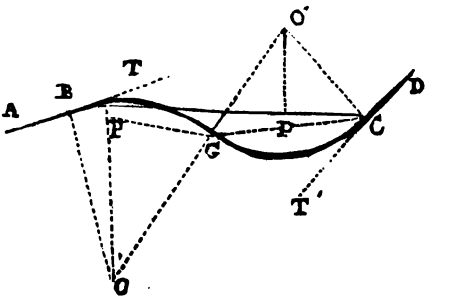
Join B C; put O B = O C = R, O′ C = O′ C′ = r, δ = B C′, α = $\angle$ T B C′, and β = $\angle$ T C′ B; then

$$r = \frac{\frac{1}{2}\delta\,(-\delta + 2\,\mathrm{R}\sin\alpha)}{\delta\sin\beta + 2\,\mathrm{R}\sin^2\frac{\alpha+\beta}{2}}$$ * Whence, having measured the distance B C′, between the given points, and taken the angles T B C′, T C′ B, the radius O′ C = O′ C = r may be readily found. In making the calculation the natural sines must be used.

Problem VIII.

To find the radii of the serpentine curve mechanically.

The serpentine curve has been defined in Art. 1., page 156. In the annexed figure B G C is a serpentine curve; A B, D C, the tangents at its extremities; B O, C O′, the radii of the respective portions B G, G C, of the curve; and O G O′ the common normal of these portions. If the curve is required to pass through or near the point G, the mechanical curves, or curve rulers, must be applied, till the portion B G of the curve is satisfactorily determined, as in Prob. VI.; after which the



* *Investigation.*—Draw O p, O′ p' $\perp$ to B C′; and through O′ parallel to B C draw O′ q, meeting O p in q. Then O O′ = R − r, B p = R sine α, O p = R cos. α, C′ p' = r sine β, O′ p' = r cos. β, O q = O p − O′ p' = R cos. α − r cos. β, and O′ q = p p' = $\sqrt{\mathrm{O'O^2} - \mathrm{O'}q^2} = \sqrt{(\mathrm{R}-r)^2 - (\mathrm{R}\cos.\alpha - r\cos.\beta)^2}$. Also δ = R sine α + r sine β + $\sqrt{(\mathrm{R}-r)^2 - (\mathrm{R}\cos.\alpha - r\cos.\beta)^2}$. By reducing this equation, and putting for sine^2 + cos.^{2} its value 1, there results $r = \frac{\frac{1}{2}\delta\,(-\delta + 2\,\mathrm{R}\sin\alpha)}{\delta\sin\beta + 2\,\mathrm{R}\sin^2\frac{\alpha+\beta}{2}}$; which is the formula for the required radius.

other portion G C may be found in the same manner as in the problem just referred to; and, having marked the tangental points B, G, C, and determined the radii B O, C O′, from the mechanical curve, the operation will be complete.

Problem IX.

1. *To find one of the radii of the serpentine curve by calculation, the other radius and tangent points being given.* (See figure to last Problem.)

By using the same notation as in Prob. VII., we shall have the following general formula for finding one of the radii of the curve when the other is given, whether the curve be serpentine or compound.

$$r = \frac{\frac{1}{2}\delta\,(\pm\,\delta \mp 2\,\mathrm{R}\,\sin\,\alpha)}{\delta\,\sin\,\beta + 2\,\mathrm{R}\,\sin^2\frac{\alpha \mp \beta}{2}}.*$$

In the present Problem the upper signs are used; and by assigning the proper values to R, δ, α, and β, the length of the required radius r will be found.

Note. It may thus be readily known whether the required radius is greater or less than 80 chains; if less, either the given radius must be, if possible, diminished, or the distance of the tangental points increased, according to the requirements of the case. Also, by joining B G, C G; and determining either of the isosceles triangles OBG, O′CG, the position of the common normal point, or point of contrary flexure of the curve, may be exactly found, for the purpose of setting out the curve on the ground.

2. *When the two radii* B O, C O′, *of the serpentine curve are equal, to find this common radius by calculation, the tangental points being given.* (See last figure.)

In this case $\mathrm{R} = r$, the other notation being the same as before; also put σ = arc to cos. $\frac{1}{2}$(cos. α + cos. β); then,

$$r = \frac{\delta}{\text{sine } \alpha + \text{sine } \beta + 2 \text{ sine } \sigma}.\dagger$$

Note. By this very concise formula the common radius may be readily determined; thus giving the most preferable method of forming the serpentine curve, when the nature of the ground will admit of its being done. It thus may be readily ascertained from the *data* whether the radius be greater, equal to, or less than 80 chains; and, if less, the distance of the tangental points must be increased, till the required length be obtained.

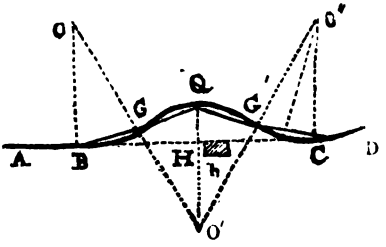
* By drawing the ⊥s Op, O′p′, and proceeding with the investigation in the same manner as in foot note to Prob. VII., the general result above referred to, will be obtained.

† This formula is a particular case of that given in Prob. VII.; its investigation is also fully given in *Baker's Railway Engineering.*

Problem X.

To make a given deviation from a straight line of railway, by three curves; that the works of the line may avoid a building or other obstruction, situated on or near to it.

1. Let A B C D be a straight portion of the railway, h a building or other obstruction on the line. Take H Q of a sufficient length for a deviation, that the line may avoid the object at h; and through Q draw a curve GQG′ of radius QO′ = to, or greater than 1 mile. Draw also, two other curves, BG, G′C, of like radius, to the first curve at G and G′, and the line at B and C: then the lines O O′, O″ O′, joining the centres of the curves, will pass through their points of contrary flexure at G and G′.



2. *Calculation.* Put r = common radius O B = O′Q = O″C, and d = required deviation = H Q; then $BH = HC = \sqrt{d(4r - d)}$, and the four equal chords B G, G Q, &c., are each = $\sqrt{dr}$.

Example.

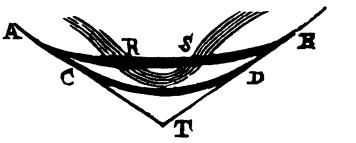
Let the deviation Q H = d = 3 chains, and the radius B O = r = 1 mile = 80 chains; then $BH = HC = \sqrt{3(320 - 3)} = 30{\cdot}84$ chains, and B G = G Q = &c. = $\sqrt{240} = 15{\cdot}49$ chains. These distances being set out will give the required points in the deviation curve B G Q G′ C, as required.

Examples of expensive severance of property by improperly setting out railway curves.

1. It has been already shewn (Prob. I.) that the curve adopted in joining two straight portions of a railway, ought to avoid, as far as possible, expensive cutting, severance, bridges, &c., provided its radius be equal to or greater than one mile, if not near a terminus or principal station. The following injudicious violations of this rule came immediately under the author's observation.

In the annexed figure, A R S B is a curve of 1½ miles radius, at Westwick, near Cambridge, in the Wisbeach, St. Ives,

and Cambridge Railway: this curve unnecessarily makes an expensive severance, by being laid out through a gentleman's park, within 100 yards of his house, and by crossing and re-crossing a brook 16 feet in width at R and S, within the space of a few chains, the brook being left by the curve only 30 yards, at the widest part between the points of crossing. Thus the line here requires two oblique bridges at R and S, which might have been avoided, as well as the expensive severance of the park, by extending the tangental portions A C, B D of the railway a few chains each, and substituting a curve C D of one mile radius to supply the place of the one thus ignorantly adopted; the ground being almost perfectly level, and the limit of deviation at the same time, admitting, nay at once suggesting, this improvement of the line, as it respects expense of construction.



2. A similarly expensive severance was unnecessarily made in the same line at Histon, about three miles from Cambridge, by laying out a curve of 3 miles radius, through cottages and gardens, all of which (being situated within three chains of the middle of the line on the north side thereof,) would have to be purchased by the railway company; besides the curve at the same place crossed two public roads within 40 yards of their junction: but had the straight portions of the line been extended at both ends of the curve, as in the former case, and a curve of 1 or 1½ miles radius been employed, instead of the 3 mile radius, both the cottages and gardens might have been passed near their southern boundary, and only one road would be required to be crossed, at a short distance from the junction of the two roads in question, the limits of deviation and the ground (being perfectly level) at once suggesting this improvement to any engineer of ordinary skill.

It is to be presumed that the head engineer of this line would rectify these expensive blunders, previous to the formation of the railway: although such things were too often overlooked or disregarded in the *Hudsonian* hurry of railway projects.

THE PRACTICE OF ENGINEERS IN THE ADOPTION OF CURVES IN VARIOUS RAILWAYS.

The practice of engineers differs very widely in the adoption of curves, some choosing curves of large radius, at great sacri-

fices of cost, and others adopting very small ones to avoid expensive cuttings, embankments, &c.

On the Great Western Railway, "the curves are in general very slight, chiefly of 4, 5, and 6 miles radius. Mr. Brunel considered that even a mile radius is not desirable, except at the entrance of a depôt, where the speed of the engines is always greatly slackened. And, except in these instances, the only deviation from this rule, which he has admitted, is in the curve, about one-fouth of a mile below one of the inclines, where the radius is three-fourths of a mile."—*Railway Magazine*, vol. I., page 418.

Mr. R. Stephenson, in his evidence on the projected Brighton Railway in 1836, stated that no curve had a less radius than 1½ miles, in the line he proposed, which he considered a most convenient radius for the high velocities required for passenger trains. Mr. Stephenson does not, however, limit the *minimum* curvatures to three-fourths of a mile, if engineering difficulties, or other considerations of a sufficiently important character, suggest the adoption of curves of smaller radius.

On the Chester and Birkenhead, Birmingham and Derby, Edinburgh and Glasgow, Arbroath and Forfar, and many others, the *minimum* radius adopted for the curves is 1 mile; and the same radius is also the *minimum* in the Birmingham and Gloucester, and the Sheffield and Manchester lines, which are curved throughout almost the whole of their lengths. The London and Birmingham line, though constructed through a very uneven country, has chiefly curves of a radius exceeding 1 mile; while the Manchester and Leeds line has curves generally of three-fourths of a mile radius, with a few considerably less.

A still greater deviation from the *minimum* limit of one mile for the radius of curves, will be found in railways where great engineering difficulties were to be encountered, especially in mineral lines. The Taff Vale Railway, which is a single line, appears from Sir F. Smith's report to have curves of the subjoined radii and length.

Chains radius.		Length of curve.
7	.....	7 chains.
10	.....	26 ———
11	.....	7 ———
12	.....	18 ———
15	several curves.....	201 ———
20	ditto	182 ———
22	ditto	173 ———

Chains radius.	Length of curve.
25	29 chains.
26	37 ———
28	21 ———
30	88 ———
40	85 ———
60	20 ———
80	120 ———

These curves were used to avoid the repeated crossing and recrossing the river Taff, and to avoid the formation of several lofty embankments. The same considerations have led to the adoption of curves of similarly small radius in other mineral lines.

Pambour's formula may be successfully adopted to assign a proper super-elevation to the exterior rail to counteract the centrifugal force, arising from high velocities of trains in curves of small radius. This formula, with the results deducible therefrom, shall be given in the last chapter of this work.

REMARKS ON THE INVENTION OF THE METHODS OF LAYING OUT RAILWAY CURVES ON THE GROUND.

No less than twelve gentlemen, in the course of the last five years, have published on the subject of laying out railway curves; their methods, at least the practical ones, being essentially the same as the four methods which I invented 27 years ago, and which were first published in the *Gentleman's Diary*, a second time in the *Ninth Edition of Nesbit's Surveying*, thirdly in *Baker's Railway Engineering*, and now the fourth time in this work. These methods having been generally adopted in practice by engineers up to the present time, as being applicable to all cases that can occur, and obviously as well adapted to practical use as the nature of the subject will admit, and consequently not likely to be superseded by any methods that have, or will be proposed, I, therefore, think it right to claim the invention of these methods. My attention was drawn to this subject at the time of the laying out of the first portion of the Stockton and Darlington Railway, and being then myself a land surveyor, accustomed to mathematical investigations, and residing near the railway in question, as well as acquainted with some of the surveyors and other gentlemen connected therewith, I communicated three of my methods, which are given in Problems II. to V., to them, some of whom were afterwards surveyors on the Liverpool and Manchester Railway, and various others that were afterwards projected.

These methods were at the same time communicated to several of my mathematical friends, some of whom are still living; among whom was Professor Leybourn, of the Royal Military College, Sandhurst, to whom I sent them, with other matter referring to the same subject, in 1824, for insertion in the Gentleman's Diary, to which, it may be seen, I had previously contributed for some years. But on account of the length of the subject, the small space then chiefly allotted to mathematical solutions in the *Gentleman's Diary*, and the very little public notice then directed to railways, my paper did not appear for some years; and, as I did not then anticipate that railways would ever assume such general importance as they now do, I paid less attention to my paper on this subject than it now appears to deserve. Besides, at the same time I obtained the prizes for answering the prize questions in the *Gentleman's Diary* and the *Gentleman's Mathematical Companion*, in the same year (1824), I was consequently induced by my friends to make preparation to enter myself, as a student of St. John's College, Cambridge. I, therefore, neglected the subject of railway curves till 1834, when I saw Professor Leybourn, who, on account of the then rising importance of railways, promised, at his earliest convenience, to insert my paper on curves, with improvements and additions, which I then gave him, either in the *Gentleman's Diary* or his *Mathematical Repository:* although Professor Leybourn was among those that then held opinions extremely hostile to the general adoption of railways, as Utopian schemes that would disappoint or ruin their projectors, and declared his willingness to insert any other of my mathematical productions, in preference to the paper in question. He was, however, acquainted with some of the leading railway engineers, and on his recommendation and that of Dr. Gregory, of the Royal Military Academy, Woolwich, I became from that time, wholly connected with railway surveys under various engineers, and parish surveys under the Tithe Commissioners for first class maps (See pages 58 to 61, where I have given the chief lines of construction of several parish maps, including those recommended by the Tithe Commissioners).

Professor Leybourn, having satisfied the claims of his other contributors, at length inserted my four methods of laying out railway curves, in November 1837, in the *Gentleman's Diary* for the following year; however, through some inadvertency, the original paper was not accompanied by the improvements and additions, above referred to. Besides, it is here proper to

add that I communicated these four methods to at least fifty gentlemen, previous to their publication.

The gentlemen that have published on this subject, after the publication of my paper, are Rankine, Heald, Castle, Law, Huntingdon, Hill, Gardner, May, Darbyshire, Charlton, Broadie, Kennedy, and Hackwood. Mr. Rankine's method has the best claims to originality; it is, however, only a modification of my method, given in Prob. IV., and possesses, moreover, the defect of my method given in Prob. II., (see Note, page 167). One of these authors recommends three of my methods, suggesting improvements on my common method by means of tedious formulæ to calculate minutiæ hardly appreciable in any curve that occurs in practice: he also recommends the offsets to be laid off radially from the tangent to the curve, which would require the centre of the curve to be found, which is well known to be impracticable in most cases, and in all cases unnecessary. A third of these authors recommends what he calls the method by "curve frames," which is theoretically little different from my common method, and has the same defect; to remedy which, he devotes five or six pages of his work to geometrical methods of making the defective curves fit one another by introducing new tangents, these methods being chiefly impracticable on the ground, and consequently of no use. Several of these gentlemen give tables of offsets, extended over from 20 to 50 pages, the use of which are at once supplied by a table of one page at the end of this work; while others give a variety of complex formulæ for calculating the offsets, which will be anything but acceptable to those who have been accustomed to lay out railway curves by my methods.

Almost all these authors have published one or more of my methods, without acknowledging from whence they obtained them, excepting what two or three of them have taken from Mr. Rankine. Isolated methods have been published by others on the same subject in periodicals, but these are either copies, or slight variations of my methods, or such as are not conveniently practicable.

As exceptions to the rule adopted by the gentlemen above referred to, two highly talented gentlemen, Professors Tate and Young, have duly acknowledged me as the inventor of these methods, the former in his Geometry, and the latter in his Mensuration.

I have now briefly given a kind of history of the invention of the methods of laying out railway curves, not meaning to infer that some other person would not have invented the same me-

thods, had I never given my attention to them; as the subject involves difficulties more of a practical than of a mathematical character, a knowledge of these practical difficulties and of the use of surveying instruments being requisite to perfect these methods. Moreover, my publications on the same subject, both in the ninth edition of *Nesbitt's Surveying* and in my *Railway Engineering*, have been well received by the public.

T. BAKER.

CHAPTER III.

ON SETTING OUT THE SURFACE WIDTHS OF RAILWAYS.

After the centre of a railway has been marked out, as directed in Chap. II., the line must again be carefully levelled, and the stumps that mark the line must be numbered and entered consecutively in the level book in a vertical column, with the corresponding depths of cuttings or embankments in a second column (see Level Book, p. 182); these depths are estimated from the formation level, which is 2 feet below the intended line of the rails; the 2 feet are afterwards to be filled up with gravel to form the permanent way.

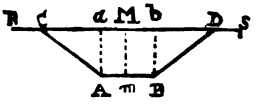
The line is now prepared for setting out the surface widths; the simplest case of which is when the surface of the ground is level as well as coincident with the formation level of the intended railway. In this case it is only required to set out half the width of the formation level on each side of the centre stump, perpendicular to the direction of the railway, adding to each half width the intended width of the side fence, and putting down stumps to mark the half widths and breadths of the fences.—When the surface of the ground is above or below the formation level, which is commonly the case, the widths must be set out by the following Problems:—

PROBLEM I.

To set out the width of a railway cutting, when the surface of the ground is laterally level, and at a given height above the formation level, the ratio of the slopes being given.*

In the annexed cross section of the cutting, R S is the horizontal surface of the ground; A B the formation level;

* The ratio of the slopes is the proportion that the batter C *a* bears to the depth A *a*. When this ratio is as 1 to 1, C *a* = A *a*; when it is as 1½ to 1,



A C, B D the side slopes; M the middle stump, and $Mm = Aa = Bb$ the perpendicular depth of the cutting. Multiply the depth Mm by the ratio of the slopes, to which add the half width A m or a M, the sum is half the surface width to be set out from M to C; after which set out R C for the width of the fence. The same operation must be repeated on the other side of M.

EXAMPLE.

Let the width of the formation level A B = 33 feet, the depth Mm of the cutting 30, the ratio of the slopes as $1\frac{1}{2}$ to 1, and the width of the side fences each 6 feet; required the width of the cutting, and the width of land included by the fences.

$30 \times 1\frac{1}{2} + \frac{3}{2}^3 = 45 + 16\frac{1}{2} = 61\frac{1}{2}$ feet $= MC = MD = \frac{1}{2}$ width of cutting; and $61\frac{1}{2} + 6 = 67\frac{1}{2} = MR = MS = \frac{1}{2}$ width of land.

The doubles of these are the whole widths.

Construction of the cross section. Draw A B = width of formation level = 33 feet; perpendicular to A B, at its middle point m, draw Mm = given depth = 30 feet; through M parallel to A B draw C D, making C M, M D each $= \frac{1}{2}$ A B $+ 1\frac{1}{2} \times Mm$; join A C, B D; then A B D C is the cross section required.

NOTE 1. The numbers in the column, marked "computed half-widths," in the level book are found by this Problem.

NOTE 2. If the figure A B D C be inverted, it will represent the cross section of an embankment; for setting out the width of which the same method obviously applies, as that just given for a cutting.

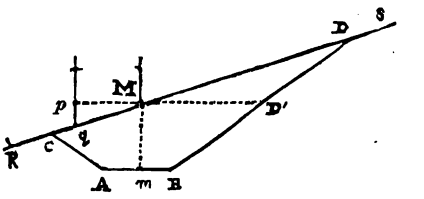
NOTE 3. Let ω = AB = breadth of the formation level, $d = Mm$ = depth of cutting or embankment, and r = ratio of the slopes. Then $1 : r :: d : dr = aC$; hence $dr + \frac{1}{2}\omega = MC = MD$; $2(dr + \frac{1}{2}\omega) = 2dr + \omega = CD$ = surface width of cutting or embankment; and $\frac{1}{2}(AB + CD) \times Mm = (dr + \omega)d$ = area of the cross section.

PROBLEM II.

The same things being given as in the last Problem, to set out the width of the cutting, when the ground is laterally sloping, the lateral fall of the ground in a given horizontal distance being also given.

Let C D be the sloping surface of the ground, A B D C the cross section of the cutting, and p D′ a horizontal line passing through the centre stump M, M D′ being the computed half

C a = $1\frac{1}{2}$ times A a. This ratio varies according to the nature of the material through which the cutting is made, being less in rocky or clayey ground, and greater in soft or sandy ground.



width of the cutting. —Fix the levelling instrument so that by turning the telescope 2, 3 or more chains of the line may be seen, on both sides of it; set up a levelling staff at M, and another at q, not exceeding a chain's distance from M, observing the level readings on both staves, the difference of which is equal to $p\ q$; measure the sloping and horizontal distances M q, M p with a tape line in feet. Then take the computed half width M D′, (found by the last Problem) and multiply it by M q, reserving the product; multiply the difference of the stave readings $p\ q$ by the ratio of the slopes, add and subtract the product to and from the horizontal distance M p, reserving the sum and difference; lastly, the reserved product, being divided by the reserved sum, will give the corrected half width M C, and by the reserved difference the corrected half width M D.

EXAMPLE.

The depth of the cuttlng at M is 22 feet, the bottom width A B = 36, the sloping distance M q = 25, the level distance M p = 24, the difference of the readings of the staves $p\ q$ = 7 feet, and the ratio of the slopes as $1\frac{1}{2}$ to 1. What are the corrected half widths M C and M D?

$22 \times 1\frac{1}{2} + \frac{36}{2} = 33 + 18 = 51$ feet=computed half width.

25

1275 reserved product.

24

$7 \times 1\frac{1}{2} = 10\frac{1}{2}$

reserved sum $= 34\frac{1}{2}$) 1275 (36·95 feet = cor. $\frac{1}{2}$ width M C.

reserved diff. $= 13\frac{1}{2}$) 1275 (94·44 feet = cor. $\frac{1}{2}$ width M D.

Construction of the cross section.—Draw B D′ M m, as in Prob. 1, ; lay off the given horizontal distance M p = 24 feet; draw $p\ q$ parallel to M m, and equal to the difference of the stave readings at M and q; through M, q draw C D, meeting A C′ in C and B D′ prolonged in D; then A B D C is the required cross section.

NOTE 1. The following general formula for the values of MC, MD, is easily remembered, and would perhaps be preferred to the rule, as given above, by those who are accustomed to the use of symbols,

$$\text{Corrected } \tfrac{1}{2} \text{ width} = \frac{a' s}{l \pm r h}.$$

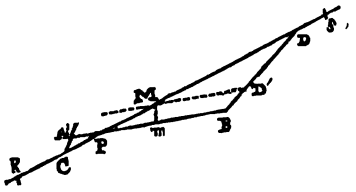
The positive sign is used for M C, and the negative for M D; M C′ being $= a'$, $p q = h$, M $q = s$, M $p = l$, and the ratio of the slopes as $r : 1$. The investigation of this formula is given in *Baker's Railway Engineering*, page 35, wherein, when h is very small, s may be taken $= l$ without error.

NOTE 2. If the figure A B D C be inverted, is will obviously represent the cross section of an embankment of like dimensions, the longer distance, in this case, being measured down, and the shorter up the slope.

PROBLEM III.

The same things being given, as in Prob. II. *to set out the width, when it consists partly of a cutting and partly of an embankment.*

In the annexed figure B D P C A is the cross section of the works of a railway, consisting of the cutting B P D and the embankment A P C; A B is the formation level, M the central stump, and D C the sloping surface of the ground. When the cutting B P D extends over more than half the formation level A B, the corrected half width M D′ is found as in Prob. II. But to find the other corrected half width M C, passing under the embankment A P C′, proceed as follows. Multiply M C by the difference of the width of the formation level A B and the estimated half width M D′, and divide the product by M D′, and the result will be the corrected half width M C.



EXAMPLE.

Let the bottom width A B = 36 feet, the depth M m = 4 feet, the ratio of the slopes as 2 : 1, and the difference of level readings 7 feet at 25 and 24 feet from the central stump, respectively estimated on the slope and horizontally; required the corrected half width M D, M C.

By Prob. I..... $4 \times 2 + \frac{36}{2} = 26$ feet = estimated $\frac{1}{2}$ width M D′.

By Prob. II..... $\frac{26 \times 25}{24 - 7 \times 2} = 65$ feet = corrected $\frac{1}{2}$ width M D.

By Prob. III... $\frac{65 \times (36 - 26)}{26} = 25$ ft. corrected $\frac{1}{2}$ width M C.

Construction of the cross section.—Proceed as in Problem II., excepting that A C must here be drawn parallel to B D.

NOTE 1. The formula for finding the corrected half width, where there is both a cutting and an embankment is

$$M C = \frac{(\omega - \omega')s}{l - rh};$$

wherein ω is the width of formation level, and the other symbols the same as in Note 1, Prob. II. See *Baker's Railway Engineering.*

NOTE 2. To find the distance from M to P, where the cutting and embankment meet, use the following proportion;

$$\text{as } pq : Mq :: Mm : MP,$$

$$\text{that is, } 7 : 25 :: 4 : \frac{25 \times 4}{7} = 14{\cdot}28 \text{ feet} = MP.$$

$$\text{or the formula } \frac{ds}{h} = MP.$$

NOTE 3. When the sloping surface of the cutting passes through the middle point of the formation level, that is, when the points M, m, and P, coincide; then $\omega' = \frac{1}{2}\omega$, and the formula in Note 1, becomes

$$MC = MD = \frac{\frac{1}{2}\omega s}{l - rh}.$$

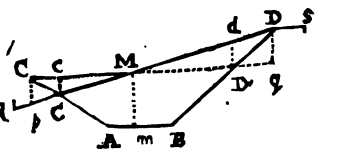
In this case the cutting and embankment are equal.

NOTE 4. By inverting the cross section of this Problem, it will at once be seen that a like calculation will be required, in the case of A P C being a cutting, and B P D an embankment.

PROBLEM IV.

To find the surface width of a cutting where the ground is very uneven.

The annexed figure is a cross section of a cutting, wherein the surface C M D of the ground is very uneven. The method of solving this Problem is by approximation, and will be best shown by an example in numbers.



Let A B = 36 feet, M m = 32, and the ratio of the slopes as 2 : 1; then M C′ = M D′ = 32 × 2 + ½ × 36 = 82 feet; lay off this distance horizontally from M to D′; then d is directly above D′. Observe the difference of level readings at M and d, which in this case is 9 feet; which being multiplied by the ratio of the slopes, that is by 2, gives 18 feet = approximate distance D d; whence M D = M d + d D = 82 + 18 = 100 feet. Again, place a level staff at D, and the reading will be found to

be 9·7 feet greater than that at M, or 9·7 — 9 = 0·7 feet greater than at d; therefore the place of the point D requires further correction, which is thus effected; 0·7 × 2 = 1·4 feet = second correction; whence M D = 100 + 1·4 = 101·4 feet; which, as the latter correction is small, may be safely assumed to be the true distance of D from M, or the horizontal distance M q. The method of finding the other corrected half width M C = horizontal distance M c, is the same as that just given, excepting that the repeated corrections are subtracted from the computed ½ width instead of being added thereto. In this manner the horizontal distance of C from M is found to be 61·8 feet.

Construction of the cross section.—Draw A B D′ C′, as in the preceding Problems; on C′ M D′, as a *datum* line lay of the reduced levels of the several undulations of the surface C M d D of the ground; (see Chap. I. p. 147,) then A B D C is the section required.

NOTE 1. When the differences of the levels at M, d and D are very great, it will require three, four, or more approximations, similar to those just given, to find each of the corrected half widths.

NOTE 2. This cross section may be inverted for an embankment, as in the preceding Problems.

LEVEL BOOK.

A level book of the following form is used in setting out the cuttings and embankments of railways.

	No. of Stumps.	Depths of Cuttings or Embankments.	Computed Half Widths.	Corrected Half Widths to Edge of Cuttings or Foot of Embankment.		Whole Widths including Fences, each 6 feet in Width.
				North.	South.	
		feet.	feet.	feet.	feet.	feet.
cutting.	183	28·00	57·00	61·80	101·40	175·20
	184	4·00	21·00	22·63	48·37	83·00
	185	30·60	60·90	64·77	71·32	148·09
embank-ment.	186	2·16	18·24	18·00	19·44	49·44
	187	16·08	39·12	40·68	58·24	110·92
	188	20·00	30·00	36·16	48·00	96·16

NOTE. The depths in the 2nd column, are found by calculation. (See Levelling, p. 153,) or by careful measurement from the sections; but the latter method is the less correct of the two. The computed half widths, in the 3rd column, are found by Prob. I.; the corrected half widths, in the 4th and 5th columns, by Probs. II., III., and IV.; and the widths in the last column are the sums of these in the 4th and 5th *plus* the breadths of the two side fences of the railway.

Problem V.

To calculate the quantity of land for a projected railway.

In preparing the estimates for a projected railway, the required quantity of land is commonly found, without respect to the lateral sloping of the surface of the ground, by taking considerable lengths of regularly rising or falling ground, in one calculation, the depths of the ends of such lengths being measured for the purpose with the vertical scale.

Rule.—Find the surface widths, fences included, from the given depths, at each end of the given length, by Prob. 1.; multiply their sum by the length in chains, and divide the product by 1320 for the content in acres.

Example.

Let the length be 16 chains, the depths of the cross sections at the ends 18 and 58 feet, the width of the formation level 36 feet, the ratio of the slopes as 2 to 1, and the width of the fences 6 feet each; required the area of the surface.

By Prob. I.

$36 + 18 \times 4 + 6 \times 2 = 120 =$ width of one end,

$36 + 58 \times 4 + 6 \times 2 = 280 =$ width of the other,

$400 =$ sum of widths.

16

132·0)640·0

The content $= 4{\cdot}84848 = 4a.\ 3r.\ 61p.$

Or, by putting ω' and $\omega'' =$ computed $\frac{1}{2}$ widths of cuttings, as found by Prob. I., $l =$ length, and $f =$ breadth of one of the fences, there will result.

$$\frac{(\omega' + \omega'' + 2f) \times l}{660} = \frac{(54 + 134 + 12) \times 16}{660} = 4{\cdot}84848 \text{ acres, as above.}$$

Problem VI.

To find the exact quantity of land for a railway.

Rule.—Take the widths, at the end of every chain, from the last column of the level book; add continually together the first and last widths, with twice the sum of all the intermediate widths, and divide the result by 1320 for the content in acres.

EXAMPLE.

Required the content corresponding to the several widths, in the preceding level book.

83·00	
148·09	
49·44	
110·92	
391·45	
2	
782·90	= twice sum of intermediate widths,
175·20	= first width,
96·16	= last width.

1054·26

1320 { 12 / 11·0

87·855

·79868 = 0*a*. 3*r*. 8*p*. the content.

NOTE. It is usual in practice to find the contents of the ground required for the works of a railway from the several proprietors, by measurement from the two chain plan, prepared for the use of the contractors. Copies are also taken from the plan, showing the positions and extents of ground required from the several proprietors for the works of the railway.

CHAPTER IV.

TUNNELLING.

1. When the depth of a railway cutting reaches 60 feet, and the ground afterwards rises rapidly for a considerable distance, the further progress of the works of the line will be the most economically conducted by making a tunnel: previous to the setting out the earthwork of which, the ground, under which it passes, must be again levelled with great care; and, if the tunnel pass beneath a very high summit, the levelling operation must be checked by the method given in Chap. I., p. 155: for, if there be the most trifling inaccuracy in the section, the gradient or gradients, on which the tunnel is proposed to be formed, will not meet at the points shewn on the section, thus greatly embarrassing the mining operation.

2. If the tunnel be formed on a single straight gradient, the

gradient must be so arranged as to incline to one of its extremities; that thus the water, generated in the tunnel, may be discharged. Strong straight poles must be firmly and perpendicularly fixed on the surface of the ground, in the proposed direction of the tunnel, one of which must be at the summit, from whence the direction of both ends of the tunnel may be observed, that the shafts from the surface to the tunnel may be sunk in the true direction. The shafts are usually sunk at the distance of four or five chains from one another, for the purpose of ventilating the tunnel, as well as for drawing the earth, &c. out of it, and for checking the accuracy of the work, during the mining operation, the depths of the shafts being determined by measurement from the section.

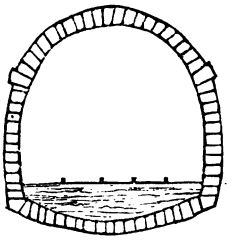
3. If the tunnel be a long one, and in springy ground, it would be advisable, if convenient, to form it upon two gradients, inclining to the extremities of the tunnel, as this arrangement would contribute much to the liberation of the water, during the mining operation, which is usually commenced at both ends of the tunnel at the same time,—but, if the work of the tunnel be required to be also commenced at the bottom of the shafts, as soon as they are sunk, which is sometimes the case, the generated water must be drawn from the shafts, or a head-way must be driven from the extremities of the tunnel to the shafts to take off the water.—A head-way, being only 5 feet in height and 3 in width, may be more readily formed than the more extensive works of a tunnel, and will be found the most efficient method of draining a tunnel in a wet or springy ground, and thus greatly facilitate the mining operation.

4. When the tunnel is required to have a curve, through a part or the whole of its direction, the curve must be carefully laid out on the surface of the ground, by one or other of the methods given in Chap. II., proper allowances being made for acclivities and declivities, and strong poles being fixed in the whole direction of the tunnel, as pointed out in Art. (2.), that the shafts may be sunk so as accurately to meet the works of the tunnel below, that their true direction may be secured, this will be more especially necessary where the tunnel is curved, though curves ought, if convenient, to be avoided in tunnels, as accidents, attributable to curves, are more dangerous in tunnels, there being less chance for escape, while assistance cannot be so efficiently given in such secluded situations.

5. If the mining operations of the tunnel be required to commence at the bottoms of the shafts, before the head-ways have been driven to them from the ends of it, the angle that the

magnetic meredian makes with the direction of the tunnel must be carefully observed at the top of each shaft, with a miner's compass, (a kind of circumferentor,) that the proper direction of the work may be set out with the same instrument at the bottom of the shaft: but, if the tunnel be curved, the direction of the tangent to the curve at the top of each shaft must be observed, that the same direction may be given to it at the bottom of the shaft, and that from thence the curve may be laid out in the tunnel, in the same direction as it was on the surface of the ground; it would be also advisable to bore from the surface to the tunnel, before the works thereof have proceeded far, to test their accuracy, as ferruginous matter in the earth might cause the magnetic needle to deviate, and thus give a wrong direction to the works, if this precaution be not taken.

6. The excavations of a tunnel on the narrow gauge should be about 30 feet in depth and width, the depth extending 5 or 6 feet below the intended line of the rails, to give room for the inverted arch and the ballasting: but when the tunnel is made through rock, sufficiently hard to form the side walls, its height need not exceed 22 or 25 feet, and its width 26 feet, the excavation in this case extending only to the formation level. For the railways on the broad guage the excavations of the tunnel must be proportionably larger.



The cross-section of the masonry of tunnel is shown in the annexed figure, with the ballasting on which the rails are laid. This cross-section is such as is required where the excavations of the tunnel is made through loose earth, the arch above being only requisite when made through hard rock.—The following notes on the construction of tunnels, are chiefly extracted from *Dempsey's Practical Railway Engineering.*

NOTE 1. Like mining and all other subterranean operations, the construction of a tunnel can be but little aided by mechanical appliances; it chiefly requires hard manual labour, exercised under circumstances which do not admit of that thorough superintendance which promotes economy, and, moreover, liable to unforeseen interruptions, of surmounting which neither the manner nor the expense can be predetermined. Thus the Kilsby tunnel, on the North Western Railway, was estimated to cost £40. per yard lineal; whereas the actual cost was £130. per yard, owing to its intersecting a quicksand that had escaped the trial borings. Thus a vast expense was incurred in setting up and working pumping machinery to dry the sand. The Box tunnel on the Great Western Railway, excavated through oolite rock, and lined with masonry only through a portion of its length, cost upwards of £100. per lineal yard. The

length of this tunnel is 3123 yards, or upwards of 1¾ miles; it has eleven principal shafts, and four intermediate ones. The Bletchingley and Saltwood tunnels, in the South Eastern Railway, cost respectively £72. and £118. per lineal yard, the greater cost of the latter work arising from the great body of water in the sand which it intersects. —The method of proceeding with tunnelling depends mainly upon the kind of material to be excavated. This having been generally ascertained by boring and trial shafts, which must be sufficiently capacious to admit readily of lowering men and materials, raising materials excavated, fixing pumps, and also for starting the headway of the intended tunnel, when the required depth is reached.

NOTE 2. The working shafts are made from 8 to 10 feet internal diameter. They are of brick work, usually 9 inches thick, and carried up 8 or 10 feet above the surface of the ground. These, and all other shafts, rest upon curbs of cast iron, fitted into the crown of the tunnel, and forming a level base for the shaft. The air shafts are of a smaller thickness and diameter, the latter of which is usually about 3 feet.—The number of working shafts will depend chiefly on the rate of speed with which the work is required to be accomplished. With plenty of men, horses, material, and plant, the work is much facilitated by sinking extra shafts, which will usually well repay their cost.

NOTE 3. The Watford tunnel, 75 chains in length, on the North Western Railway, was worked with six shafts, about 8 feet internal diameter; the brick work was moulded to fit the circumference of the shafts, and laid in two half-brick rings. Air shafts were sunk at about 2½ chains distance on each side of each working shaft. The arch and side of the tunnel were chiefly made two bricks thick, and the invert one and a half brick, except where the stratum passed through seemed to suggest an increased or diminished thickness. The form of the top of the tunnel is nearly semi-circular, supported by curved side walls standing on stone footings, or skew backs, which rest on the invert.

NOTE 4. In commencing the work of the Saltwood tunnel, referred to in Note 1., great difficulty was encountered from the great quantity of water in the green sand, which the tunnel intersects. The course adopted was to make a headway 5 feet high and 4 wide quite through the hill, on a level with the bottom of the tunnel, in which the water was collected and drained off before the tunnel was begun. The size of this, and also the Bletchingley tunnel, is 24 feet wide at the broadest part, 30 feet including the side walls; 25 feet high in the clear, 30 feet including the invert and top arch, and 21 feet above the level of the rails. The brick work of the top arch and walls varies from 2½ to 4 bricks in thickness; and the invert is 3 bricks in thickness.

NOTE 5. The section of the Box tunnel, already referred to, was designed to be 27½ feet wide at the springing of the invert, and 30 feet wide at a height of 7½ feet above this, and the clear height above the rails 25 feet. As a great portion of the tunnel is constructed by mere excavation, and without masonry, these dimensions are, in some cases, departed from, in order to clear away loose portions of the stone, and secure solid surfaces. Where brick work is used, the sides are seven half brick rings in thickness, the arch six, and the invert four. During the construction, the constant flow of water into the works, from the numerous fissures of the rock, compelled pumping on a most expensive scale to be adopted.

NOTE 6. When much water issues from the earth, in sinking shafts or building tunnels, the back of the wall should be well lined with puddle, and roman cement should be used instead of mortar. The entire tunnel at Kilsby was built with roman cement, the thickness of the brick-work being mostly 27 inches. This tunnel is 2423 yards long: it has two ventilating shafts 60 feet

in diameter; the brick-work of which is 3 feet thick, and laid in roman cement throughout: these shafts intersect the line of the tunnel, and form curved recesses, by part of their width extending beyond the tunnel on both sides.

CHAPTER V.

THE METHODS OF FINDING THE CONTENTS OF RAILWAY CUTTINGS, &c.

In making the estimates for a projected railway, the contents of the several cuttings, embankments, &c., are in most cases found by tables, calculated for the purpose, the surface of the ground being considered as on the same level as the centre of the line. But when the projectors of the line have been empowered to construct it, cross sections of the cuttings are carefully taken, at every variation of the surface of the ground, especially if the surface be sidelaying, or inclined laterally with respect to the direction of the line, as in the cross sections in Problems II., III., and IV., Chap. III. The distances of these cross sections may vary from 10 or 12 chains to less than 1 chain, according to the regularity or irregularity of the slopes of the surface. These cross sections must next be plotted on a large scale, as in the Problems just referred to, and their areas found by any of the methods given in Chap. III., preparatory to finding the contents of the cuttings by tables; but if the surface lines of any of the cross sections be level, or so nearly so as to be readily reduceable to a level surface line by casting, their areas need not be found, their depth only being required for finding the contents by the tables.

NOTE. Some take a mean of the areas of every two consecutive cross sections, and others a mean of the depths, where the surface line is level, as a basis for calculating the contents of the cuttings, *which methods are both erroneous, especially where the consecutive areas or depths of the cross section differ considerably.*

TABLES FOR FINDING THE CONTENTS OF RAILWAY CUTTINGS, &c.

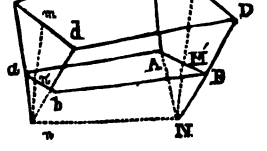
Numerous tables exist for this purpose, some of which are voluminous; those by M'Neill, Bidder, Huntingdon, Hughes, Bashforth, Sibley and Rutherford, Law and Lowe, are well adapted for the purpose, assuming the surface line of the cross sections to the level, or to be reduced to that position; but none of these tables will properly apply to sectional areas, which is the most important part of their use, excepting Bashforth's; *but his method of using them is erroneous, the error approximating to*

50 *per cent, as a maximum.* I would therefore recommend for this purpose, the *General Earthwork Table,** in conjunction with *Two Auxiliary Tables,* on the same sheet, in *Baker's Engineering,* as being applicable to all varieties of ratio of slopes and widths of formation level in common use; and with the help of *Barlow's Tables of Square Roots,* these tables will apply to sectional areas, with all the mathematical accuracy that can be attained, with very little more calculation than adding the contents between every two cross sections, as given by the General Table. —The contents in the General Table are calculated to the nearest unit, as are also those in the Auxiliary Table, No. 2, which is for the decimals of feet in the depths. The Auxiliary Table, No. 1, shows the depths of the meeting of the side slopes below the formation level, with the number of cubic yards to be subtracted from the contents of the General Table for each chain in length, for eight of the most common varieties of ratio of slope.

NOTE 1. These Tables, with very little additional calculation, may be extended to every variety of formation level and ratio of slopes that can occur, and even to cases where that ratio differs in the two sides of the same cutting, as shall be shewn in the following Problems.

NOTE 2. The investigations of the method of forming these tables and using them are given in *Baker's Railway Engineering,* also further investigations are given, at the end of the following Problems, respecting *Mr. Bashforth's Erroneous Methods of Calculating Earthwork.*

The following diagrams and explanations will further illustrate the method of taking the dimensions of railway cuttings, preparatory to using the above named tables.

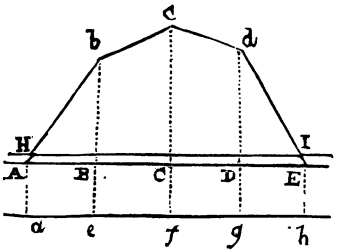


Let A B D C *c a b d*, be a railway cutting, of which A B D C, *a b d c* are the cross sections, A B = *a b* = width of formation level, M M', *m m'* the middle depths of the two cross-sections; the side-slopes A C, B D, *a c*, *b d*, when prolonged two and two, will intersect at N and *n*, at which points the prolongations of M M', *m m'* will also meet, thus constituting a prism A B N *n a b*, the content of which is to be deducted from the whole content, given by the General Table, by means of the Table No. 1.; in which the depth M'N = *m'n* is also given, as already stated, to several varieties of slope and bottom width.

To place this subject in a more practical point of view, let

* The numbers for the side slopes, forming the alternate lines in *Bidder's Table,* will supply the place of the General Table.

the annexed figure represent a longitudinal and vertical section of a cutting, passing through the middle A E of the formation level. H I, the line of the rails, and *a h*, the line in which the slopes, if prolonged, would meet. It will be seen that the cutting A *b c d* E commences and runs out on the formation level A E, and that the depth A *a* = B *e* = C *f* = &c. is to be added to the several depths B *b*, C *c*, D *d* of the cutting, the first and last depth at A and E being each = 0; or, what amounts to the same thing, the several depths must be measured from the line *a h*: thus, A *a*, *b e*, *c g*, &c. are the depths to be used. And since the depth A *a* is given in Table No. 1, for all the most common cases, or it may be readily found by calculation for all cases, as shall hereafter be shown, the line corresponding to *a h* must, therefore, be ruled on the railway-section, at the proper distance below A E, from which the several depths must be measured; or the vertical scale may be marked with Indian ink (which may be readily rubbed off) at the same distance, and this mark may then be applied to the formation level A E, for the purpose of measuring the several depths.—In the case of an embankment, the line for the several depths must be placed at a like distance above the formation-level.



Problem I.

The several depths of a railway cutting to the meeting of the side slopes, its width of formation level, and the ratio of the slopes being given, to find the content of the cutting in cubic yards, from the Tables referred to, the distances of the depths being one chain each.

Rule.—Take the several quantities, corresponding to every two succeeding depths of a cutting or embankment, measured to the meeting of the side slopes, at the distance of 1 chain each, from the General Table in *Baker's Railway Engineering* and multiply their sum by the ratio of the slopes; from the product subtract the cubic yards, corresponding to the given bottom width and ratio of slopes from Table No. 1., multiplied by the whole length of the cutting, and the remainder will be the content of the cutting in cubic yards.

But, when the distances of the depths are greater or less than 1 chain, the quantities of the General Table must be multiplied by their respective distances.—And, when the distances are given in feet, the quantities must be multiplied by those distances, and the final result divided by 66 for the content in cubic yards, as in the following examples.

EXAMPLES.

1. Let the depth of the railway cutting or embankment to the meeting of the side slopes, at the end of every chain, be as in the following table, the bottom width 30 feet, and the ratio of the slopes as 2 to 1; required the content in cubic yards.

Dist. in chains.	Depths in feet.	Qnts. per G. Table.
0	10	
1·00	33	1238
2·00	39	3175
3·00	35	3350
4·00	10	1365
For slope 1 to 1 .		9128
		2
2 to 1 .		18256
Subtract 275 × 4		= 1100
Content in cubic yds.		= 17156

NOTE. In the annexed table the quantity 1238, corresponds to the depths 10 and 33 feet, in the General Table; the quantity 3175 to the depths 33 and 39, and so on for the succeeding depths. By the Auxiliary Table No. 1. it will be seen, that the depth to be added below the formation level, for the given width and ratio of slopes, is 7·50 = 7½ feet, therefore, the cutting begins and ends with a depth of 10 − 7½ = 2½ feet. The corresponding number of cubic yards, to be deducted for each chain in length, is multiplied by 4 chains, the whole length of the cutting, thus giving the whole quantity to be deducted, the remainder being the true content in cubic yards of the cutting.

2. The several depths of a railway cutting to the meeting of the side slopes are as in the annexed table, the bottom width being 30 feet, and the ratio of the slopes 1½ to 1; required the content of the cutting.

Dist. in chains.	Depths in feet.	Products for Dist. greater than 1 chain.	Total quantities.
0	10		
1·00	16	. . . .	420
2·00	20	. . . .	795
4·00	25	1243 × 2	2486
5·00	32	. . . .	1996
7·00	39	3091 × 2	6182
8·00	45	. . . .	4319
10·00	50	5520 × 2	11040
12·00	40	4971 × 2	9942
13·00	30	. . . .	3015
14·46	10	1059 × 1·46	1546
For side slopes		1 to 1	41741
		½ to 1	20870
		1½ to 1	62611
		366·67 × 14·46 =	5302
		Content in cubic yards	57309

NOTE. When any of the distances between two succeeding depths is greater or less than 1 chain, the corresponding quantity from the General Table must be multiplied by that particular distance; as the distances between the depths 20 and 25, and between 32 and 39, &c. the distances being 2 chains. The last distance, viz., that between 30 and 10, is 1·46; in this case 2 figures must be cut off for decimals, after multiplying.

Dist. in feet.	Depths in feet.	Quantities multiplied by length.	Total quantities.
0	37		
90	50	4660 × 90	419400
178	61	7554 × 88	664752
278	39	6210 × 100	621000
For slopes 1 to 1			1705152
——— ½ to 1			852576
——— 1½ to 1			2557728
		366·67 × 278 =	101933
			66)2455795
Content in cubic yards			37209

3. Let the depths of a railway cutting to the meeting of the side slopes, and their distances in feet be as in the annexed table, the bottom width 30 feet, and the ratio of the slopes 1½ to 1; required the content in cubic yards.

NOTE. When the distances are in feet the quantities from General Table must be respectively multiplied by their distances, the quantity from Table No. 1 by the whole distance, and the result divided by 66, the feet in 1 chain, for the content in cubic yards, as in the annexed operation. See *Baker's Railway Engineering.*

PROBLEM II.

CASE I.—*The areas of two cross sections of a railway cutting to the intersection of the side slopes, its length in chains, bottom width, and ratio of the slopes are given; required the content of the cutting in cubic yards.*

RULE.—With the square roots of the given areas, as depths, find the content from the General Table, as in the last Problem, from which subtract the quantity answering to the given width, and the ratio of sides slopes from Table No. 1, and the remainer, being multiplied by the length, will be the content required.

NOTE. If the length be given in feet, proceed as in Example 3, last Problem.

EXAMPLE.

1. Let the two sectional areas of a cutting be 4761 and 1296 square feet, the bottom width 36 feet, the length 3·25 chains, and the ratio of the side slopes 2 to 1; required the content in cubic yards.

$\sqrt{4761} = 69$
$\sqrt{1296} = 36$ } content per General Table ... 6959

For bottom width 36 and slopes 2 to 1 per Table No. 1 396

Content for 1 chain in length 6563

3¼

19689
1641

Content for 3·25 chains ....21330 cubic yds.

CASE II.—*To find the content when the depths are given in feet and decimals of feet.*

RULE.—Let a and b be the feet in any two succeeding depths, and α and β their respective decimal parts; find the quantity answering to a and b from General Table, as in the former cases; then,

$2a + \beta$, rejecting the last figure, and α will shew the number to be added in Auxiliary Table No. 2, and

$2b + \alpha$, rejecting the last figure, and β will show the number to be added in the same Table.

After which deduct for the quantity below the formation level, as before.

EXAMPLE.

Let the sectional areas be 1406 and 2560 square feet, the bottom width 36 feet, length 4 chains, and ratio of slopes $1\frac{1}{2}$ to 1; required the content in cubic yards.

Here $\sqrt{1406} = 37{\cdot}5$ and $\sqrt{2540} = 50{\cdot}4$. Put $37 = a$, $50 = b$, $\cdot5 = \alpha$, $\cdot4 = \beta$; then the depths a and b, per General Table, give 4660

$2a + b = 124$, or 12, (by rejecting last figure) and $\beta = \cdot4$, per Table No. 2, give 39

$2b + a = 137$, or 14 nearly, (by rejecting last figure) and $\alpha = \cdot5$ give 57

4746

For bottom width 36 and ratio of slopes $1\frac{1}{2}$ to 1, deduct 528

Content for 1 chain in length 4218

4

Ditto for 4 chains in length ..... 16872 cubic yds.

CASE III.—*In measuring contract work, where great accuracy is required, the $\frac{1}{000}$ ths of a foot, or second decimals, must be used in the calculation, by taking for them $\frac{1}{10}$th of their respective quantities in Table No. 2.*

EXAMPLE.

The areas of seven cross sections of a railway cutting to the meeting of the side slopes, and their distances are as in the annexed table; the bottom width is 30 feet, and the ratio of the slopes $1\frac{1}{2}$ to 1; required the cubic yards in the cutting.

Dist. in Chains.	Areas in sq. feet.
0	2727
2·00	3136
6·00	4221
9·60	4100
14·00	5141
16·00	3759
18·00	2161

Ans. The content, per General Table, and Table No. 2, is 172318 cubic yards, from which the quantity corresponding to the

given bottom width and ratio of slopes × by the whole length, viz. 275 × 18 = 4950 cubic yards, must be deducted, which leaves 167568 cubic yards, the content required.

NOTE 1.—When the distances of the sectional areas are given in feet, the quantities of the General Table must be multiplied by their respective distances, and the final result divided by 66, as in Example 3. Prob. I.

NOTE 2.—When the surface lines of the sectional areas are either level or are readily reducible to that position, the decimals, if any, in depths must be taken into the calculation, as in Cases II. and III.

PROBLEM III.

To adapt the General Table to such widths of the formation level and ratios of slope as are not found in Table No. 1.

Put $\omega = \frac{1}{2}$ width of formation level, and $r : 1$ the ratio of slope.

Then $\frac{\omega}{r}$ = feet to be added to the depth of cutting below formation level.

And $\frac{22\,\omega^2}{9r}$ = cubic yards to be subtracted for each chain in length.

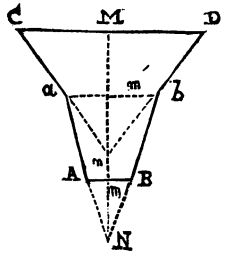
EXAMPLE.

Let the width of formation level be 26 feet, and the ratio of slopes $1\frac{1}{4}$ to 1; then $\omega \div r = 13 \div 1\frac{1}{4} = 10{\cdot}4$ ft. = distance below formation level to meeting of slopes.

And $22 \times 13^2 \div 9 \times 1\frac{1}{4} = 330{\cdot}5$ cubic yards to be deducted for each chain in length from the contents of the General Table.

PROBLEM IV.

To find the content of a cutting, when each of the sides have two different ratios of slope.



RULE.—When the cutting A B C D has two different slopes, as A *a* or B *b* and *a* C or *b* D; it must be divided into two parts by the line *a b*, and the quantities of the parts A B *b a*, *a b* D C of the cutting must be found separately by Prob. I. If the surface line C D be sloping or curved, Prob. II. will also be required; and, if the depths *m' n'*, *m* N and their corresponding quantities of cubic yards are not found in Table No. 1, they must be found by Prob. III., the sum of the contents of the two parts being the required content of the cutting.

NOTE. Cuttings of this kind are often advantageously adopted, where their upper parts are of loose or springy earth, and their lower parts strong clay or rocky.

PROBLEM V.

To find the content of a cutting when the ratio of the slopes of the two sides are different.

CASE I.—*When the surface is level.*

RULE.—Find the central depths to the meeting of each of the side slopes, and take their corresponding contents by Prob. I. for the whole length of the cutting, in the same manner as if it were for two cuttings, and, from the sum, subtract the sum of the cubic yards corresponding to the given bottom width and ratios of slope multiplied by the whole length of the cutting, and half the difference will be the content required.

CASE II.—*When sectional areas are given.*

RULE.—Find the contents corresponding to the sectional areas to the meeting of the side slopes by Prob. II., and from their sum deduct $\frac{44\ \omega^2\ l}{g(r+r')}$ cubic yards for the content. In this formula $\omega = \frac{1}{2}$ bottom width, $l =$ whole length of cutting, and r and r' the first terms of the ratios of the side slopes.

NOTE. Cuttings with slopes of this kind are frequently adopted in practice, where the ground is springy on one side of them, the greater ratio of slope being on the springy side.

PROBLEM VI.

To find the quantity of the cutting of a tunnel.

RULE.— Multiply continually together the width, mean height and length; divide the product by 9, if the length be given in yards, but, if the length be given in chains, multiply the product by 22 and divide by 9, the width and height, in both cases, being given in feet.

NOTE. Examples are not given in the three last Problems, the methods of solving which being sufficiently obvious from the Rules and the first three Problems.

THE FOLLOWING EXAMPLES SHOW THE ERRORS OF METHODS PUT FORTH BY MR. BASHFORTH AND OTHERS, TO FIND THE CONTENT OF CUTTINGS.

EXAMPLES.

1. The areas of two cross sections of a cutting to the meeting of the intersection of the side slopes, are 1296 and 361 square feet, their distance one chain, the bottom width 36 feet, and the ratio of the slopes 1 to 1; required the content of the cutting by the true and the erroneous methods practically used.

(1.) *By the true method:—*

$\sqrt{1296} = 36$
$\sqrt{361} = 19$ } content by General Table 1907
Deduction from Table No. 1 792

True content for 1 chain in length ..... 1115 cubic yds.

(2.) *By Mr. Bashforth's method:—*

By Table No. 1., the depth of the meeting of the side slopes below formation level is 18 feet; hence the area of the triangle below it is $36 \times 9 = 324$ square feet, which, being taken from the given sectional areas, will give 972 and 37 square feet for the areas used by Mr. Bashforth; whence,

$\sqrt{976} = 31{\cdot}18$
$\sqrt{37} = 6{\cdot}08$ } content by General Table, &c., 975 cubic yds.
This content taken from the true content, given above, leaves ..... 140 error in [defect.
Whence 975 : 140 : : 100 : 14·36, or *above* $14\frac{1}{3}$ *per cent. the error in defect by Mr. Bashforth's method.*

2. Let the areas of the two cross sections to the intersection of the slopes be 324 and 2916, and the other things as in the last example, required the content of the cutting by the correct and erroneous methods.

(1.) *By the true method:—*

$\sqrt{2196} = 54$
$\sqrt{324} = 18$ } content by General Table 3432
Deduction from Table No. 1. 792

True content for 1 chain in length ..... 2640 cubic yds.

(2.) *By Mr. Bashforth's method:—*

Here the sectional arcs used by Mr. B. are 2592 and 0, whence $\sqrt{2592} = 50{\cdot}91$, $\sqrt{0} = 0$ } content by Gen. Table 2112 nearly, which taken from the true content leaves ..... 528 { error in defect.

Whence 2112 : 528 : : 100 : 25 *per cent. the error in defect in this case.*

By taking a mean of the areas in example 1.

$\frac{1}{2}(972 + 37) \times \frac{22}{9} = 1223\frac{1}{9}$ cubic yards, *which exceeds the true content* $118\frac{2}{9}$ *cubic yards, being about* $9\frac{2}{3}$ *per cent. in excess.*

By taking a mean of the areas in example 2.

$\frac{1}{2}(2592 + 0) \times \frac{22}{9} = 3168$ cubic yards, *which exceeds the true content by* 528 *cubic yards, being* $16\frac{2}{3}$ *per cent. in excess.*

NOTE. By taking a mean depth the error in defect is just half the preceding one, or $8\frac{1}{3}$ per cent. These methods only give a near approximation to the true content, when the sectional areas of a cutting are nearly equal. See the results of the following formula and remarks.

The above examples shew with sufficient clearness the inaccuracy of Mr. Bashforth's method; but since it has been defended, *as sufficiently correct for practical purposes*, the following formula will more rigidly prove its failure. Let A′ and B′ be the sectional areas of a cutting, as used by Mr. Bashforth, and a the area between the formation level and the meeting of the slopes; then by *Baker's Railway Engineering*, page 55, $\frac{1}{81}(\sqrt{\overline{A' + a} \times \overline{B' + a}} - \sqrt{A' \times B'} - a) =$ his error in defect, the content by his method being $\frac{1}{81}(A' + B' \times \sqrt{A' + B'})$, the length being unity; whence

$$\frac{100(\sqrt{\overline{A' + a} \times \overline{B' + a}} - \sqrt{A' \times B'} - a)}{A' + B' + \sqrt{A' \times B'}} = \text{error per}$$

cent. by Mr. Bashforth's method.

EXAMPLES.

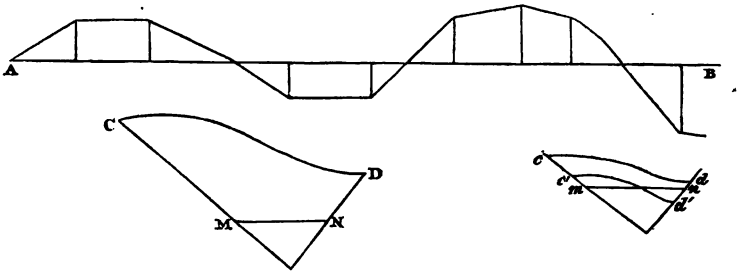
1. When $A' = \frac{1}{9}\,a$ and $B' = 9\,a$, the above formula gives $\frac{100(\sqrt{\frac{1}{9} \times 10} - \sqrt{\frac{1}{9} \times 9} - 1)\,a}{(\frac{1}{9} + 9 + \sqrt{\frac{1}{9} \times 9})\,a} = \frac{\frac{10}{3} - 2}{10\frac{1}{9}} = \frac{1200}{91} = 13\frac{1}{6}$ *per cent. error in defect.*

2. When $A' = 0$ and $B' = 3\,a$, the above formula gives $\frac{100(2 - 0 - 1)}{3} = 33\frac{1}{3}$ *per cent. the error in defect.*

3. When $A' = 0$ and $B = a$, *the error in defect is* $41\frac{2}{3}$ *per cent.*

Errors of this kind will repeatedly occur in practice, where cuttings and embankments are made as in the following figure; wherein A B is the formation level, and the corresponding curved line is a section of the earth's surface; the small figures below being cross sections taken to the intersection of the slopes, in which M N, $m\,n$, represent the formation level, the sectional areas, used by Mr. Bashforth, being those above the lines M N, $m\,n$, the cross section $c\,d\,m\,n$ may either be very

small, as in Example (1.), or 0, as in Examples (2.) and (3.); in which latter cases the surface $c'\,d'$ of the section cuts $m\,n$.



From the above figure it will at once be seen that more than one half of the sectional parts of the cuttings and embankments will be affected by errors in defect of an average of the above named per-centages, which are most decidedly prominent; and sufficient, in many cases that occur in practice, to take away more than the whole of a contractor's profits.—The error of Mr. Bashforth's method attains its *maximum, which is* 50 *per cent.*, when both the sectional areas A′, B′, vanish, which, however, is a case that can never occur in practice.

REMARKS ON MR. BASHFORTH'S AND THE AUTHOR'S METHODS OF FINDING THE CONTENTS OF CUTTINGS.

Notwithstanding the astounding errors, just proved to exist in the method given by Mr. Bashforth, for finding the contents of cuttings, he and his abettors have had the assurance to blame the author of this work for copying in his *Railway Engineering*, his (Mr. B.'s) method. Now, had Mr. B.'s method been copied by the author, he must also have copied his errors, that is, he ought to have adopted Mr. B.'s method of finding the contents of cuttings from sectional areas, where the surface of the ground is laterally sloping or curved, in which method Mr. B. entirely leaves the prism below the formation level out of the question, which the author has included in his calculations by the General Table, and afterwards deducted. For Mr. B. says (Art. 15. p. 11. of his work), "The only way to proceed in such a case is,—find the area of the cross sections in square feet, take out of Barlow's Tables the square root of each, and treat these square roots precisely as if they had been measured heights, excepting that there will be nothing to be deducted for the prism, and no multiplication for the slopes, these having been already accounted for in finding the areas."—Mr. B. then

gives an example in accordance with these directions, in which all the cross sections are large, and not greatly differing in area; the error in the result of which is, therefore, not so prominent as those in examples 1, 2, page 196, and in example 1, 2, 3, page 197, where the areas of the cross sections differ considerably.

Besides, in reference to his method of finding contents from sectional areas, Mr. B. elsewhere says, "In the case of contract estimates, numerous cross sections ought to be taken; and it matters little whether we start from the intersection of the slopes or the formation level." *The errors shewn in* page 197, *show that "it matters" so much as 25 per cent. on an average, where one of the sectional areas is moderately large, and the other either small, or tapering to* 0, *after the manner of a wedge.* Besides, "numerous cross sections" are by no means required, as Mr. B. recommends, where the surface of the ground is laterally sloping, or curved like a cylindrical surface, so that cross sections taken at from 3 to 10 chains distance to the intersection of the slopes, may be considered to be similar, or so nearly so as not to induce error; which cases very frequently occur in practice, and may be readily determined by the eye. Moreover, "numerous cross sections" are attended with considerable expense, and, for this reason alone ought to be avoided when they are not wanted, excepting where Mr. B.'s erroneous method of finding contents is used, since the amount of error will thus be diminished.

Moreover, the author's method of finding the contents in question was communicated to several gentlemen long before the publication of Mr. B.'s method, among whom was Professor Tate, of the Training Institution, near Twickenham, who is well able to judge of such matters, and who has himself published on the same subject in his *Geometry*. As for the General Table, it is calculated from one of the same formulæ, which is used by Sir John McNiell, Mr. Bidder, and others, as part of the basis of their Tables, and must, therefore, be common property, — the above named gentlemen, Mr. B., and the author, all using the results in a different manner: besides, the author has given two Auxiliary Tables to facilitate his calculations. It now remains for Mr. B. and his friends to re-examine the investigations of the methods in question, and before they pronounce Mr. B.'s method to be correct, it is their duty first to prove the author's to be false; which he now calls upon them to do. They must at the same time recollect that the length of a cutting does not affect the accuracy of these results, and that

their vague assertions, grounded on such matters, will not invalidate the demonstrations given by the author in his *Railway Engineering*, in the least degree.

CHAPTER VI.

PROBLEMS AND FORMULÆ OF UTILITY IN LAND AND ENGINEERING SURVEYING.

The areas of triangles, &c., may be expeditiously found by the following Problems, without mapping them for the purpose of finding perpendiculars, &c.

Problem I.

To find the area of a triangle, when two of its sides and their included angle are given.

Let a and b be the two sides of the triangle, α their included angle, and A the area; then

$$A = \tfrac{1}{2} a b \alpha$$

This Problem may be solved by logarithms, by which the multiplication of large numbers is avoided, and by which the area is found in acres and decimals; thus,

$$\log. A = \log. a + \log. b + \log. \text{sine } \alpha - 15{\cdot}30103.$$

EXAMPLE.

The two sides of a triangle are 1920 and 1152 links, and their included angle 53° 8′; required the area.

log. 1920.....3·28330
log. 1152.....3·06145
log. sine 53° 8′.....9·90311

16·24786
15·30103

log. A = 8·848.....0·94683

or, A = 8*a*. 3*r*. 12*p*., the area required.

Problem II.

To find the area of a triangle, when two of its angles and their included side are given.

Let α and β be the two angles, and a their included side; then

$$A = \frac{a^2 \text{ sine } \alpha \text{ sine } \beta}{2 \text{ sine } (\alpha + \beta)}.$$

By logarithms, for the area in acres,

$$\log. A = 2 \log. a + \log. \text{sine } \alpha + \log. \text{sine } \beta - \log. \text{sine } (\alpha+\beta) - 15{\cdot}30103.$$

EXAMPLE.

A side of a triangle is 2000 links, and its adjacent angles 60° 14′ and 59° 46′; required the area by logarithms.

Ans. 13*a.* 1*r.* 12*p.*

PROBLEM III.

To find the area of a triangle when the three sides are given.

Let a, b and c be the sides, and s their half sum, then

$$A = \sqrt{s\,(s-a)\,(s-b)\,(s-c)}.$$

By logarithms, for the area as above,

$$\log. A = \log. s + \log.(s-a) + \log.(s-b) + \log.(s-c) \div 2 - 5.$$

EXAMPLE.

The three sides of a triangle are 3050, 2520 and 2040 links, required the area by logarithms. *Ans.* 25*a.* 2*r.* 4*p.*

PROBLEM IV.

To find the area of a trapezium, when its four sides are given, two of its opposite angles being together = 180°.

Let a, b, c, and d be the four sides, and s = half their sum; then, $A = \sqrt{(s-a)\,(s-b)\,(s-c)\,(s-d)}$.

The method of solving this Problem by logarithms is sufficiently obvious from Problem III.

PROBLEM V.

To find the area of a trapezium when its two diagonals and the angle of their intersection are given.

Let D and Δ be the two diagonals, and α the angle of their intersection; then

$$A = \frac{D\,\Delta\,\alpha}{2}.$$

The method of solving this Problem by logarithms is the same as that given in Problem I.

NOTE. The investigations of the Formulæ used in these Problems are given in various works of Analytical Trigonometry.

PROBLEM VI.

A B C is a triangle, in which the base A B *and a point* D *therein are given,* D C *is a quality line, making a given angle with* A B; *it is required to determine* C D *so that the triangle* A B C *may contain land of a given value.*

Put A D = a, D B = b, C D = x, l = square links in an acre, sine ∠D = α, the values of the land adjacent to a and b

respectively, m and n per acre, and V the given value; then the areas of the triangles A C D, B C D are respectively $\frac{a\,a\,x}{2l}$ and $\frac{b\,a\,x}{2l}$, and their values are $\frac{a\,a\,m\,x}{2l}$ and $\frac{b\,a\,n\,x}{2l}$, whence

$$\frac{a\,a\,m\,x}{2l} + \frac{b\,a\,n\,x}{2l} = \mathrm{V}, \text{ and}$$

$$x = \frac{2\,\mathrm{V}\,l}{a\,(a\,m + b\,n)} = \mathrm{C\,D};$$

whence the position of the point C becomes known.

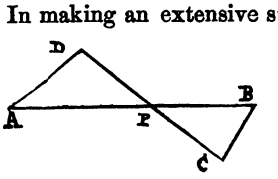
Cor. When C D is perpendicular to A B, $a = 1$, whence the above formula becomes

$$x = \frac{2\,\mathrm{V}\,l}{a\,m + b\,n} = \mathrm{C\,D}.$$

Note. This Problem would be available in solving Case IV, of Problem V, page 120; however, it is there proposed to be solved by assumed or "guess lines," conformable to the old practice of surveyors.

Problem VII.

In making an extensive survey, the fundamental lines A B, B C, C D, D A were measured, and the distance C P noted; but the distance A P was accidentally omitted. It is required to lay down the lines independent of this distance; and give a solution when A D = D P.



Put A B = a, B C = b, C P = c, P D = d, D A = e, and A P = x; then P B = $a - x$; and by trigonometry cos. $\angle$ A P D = cos. $\angle$ B P C = $\frac{x^2 + d^2 - e^2}{2\,d\,x} = \frac{(a-x)^2 + c^2 - b^2}{2\,c\,(a-x)}$; whence

$$x^3 - \frac{a(c+2d)}{c+d}\,x^2 + \frac{d(a^2-b^2+c^2)+c(d^2-e^2)}{c+d}\,x - \frac{ac(d^2-e^2)}{c+d} = 0,$$

a cubic equation, from which the value of x may be found, which determines the distance A P.

When A D = D P, that is, when $d = e$, the above equation becomes

$$x^2 - \frac{a\,(c-2\,d)}{c+d}\,x + \frac{d\,(a^2 - b^2 + c^2)}{c+d} = 0,$$

a quadratic equation, from which the value of x = A P may be readily found.

Investigation of formulæ. — As the formulæ for dividing land, at pages 118 and 120, Cases II. and III., Problem V.,

are given without investigation, to obviate all doubts of their accuracy, the processes of deducing them are here given.

(1.) By referring to the definition of the symbols, and to the diagram at page 118, Case II., it will be seen that the right angled triangles W S s, W U u, W F b, are similar, therefore,

$$s : x :: a : \frac{a\,x}{s} = \mathrm{S}\,s,$$

$$s : x :: a + b : \frac{(a+b)\,x}{s} = \mathrm{U}\,u;\ \text{hence}$$

the area in acres of W S $s = \frac{1}{2}$ W S · S $s = \dfrac{a^2\,x}{2\,s\,l}$,

——————— of S s U $u = \frac{1}{2}$ (S s + U u) S U $= \dfrac{(2a+b)\,b\,x}{2\,s\,l}$,

——————— of U u F $b = \frac{1}{2}$ (U u + F b) U F $= \dfrac{(2a+2b+c)\,c\,x}{2\,s\,l}$

Then these areas, being multiplied by their respective values per acre, and their sum equated to the required value V′, give

$$\frac{a^2 m\,x}{2\,s\,l} + \frac{(2\,a+b)\,b\,n\,x}{2\,s\,l} + \frac{(2\,a+2\,b+c)\,c\,o\,x}{2\,l\,s} = \mathrm{V}',$$

whence $x = \dfrac{2\,s\,l\,\mathrm{V}'}{a^2 m + (2a+b)bn + (2a+2b+c)co} = \mathrm{F}\,b.$ Q.E.I.

Cor. I. If there be only two qualities of land to be thus laid out, c must be made to vanish in the above formula, whence it becomes

$$x = \frac{2\,l\,s\,\mathrm{V}'}{a^2 m + (2a+b)bn} = \mathrm{F}\,b.$$

Cor. II. If there be four different qualities of land, let the breadth of the additional quality be d, and its value p; then the formula becomes

$$x = \frac{2\,l\,s\,\mathrm{V}'}{a^2 m + (2a+b)\,b\,n + (2a+2b+c)\,co + (2a+2b+2c+d)\,dp} = \mathrm{F}\,b,$$

the law of extension being sufficiently clear.

(2.) By referring to the diagram at page 119, Case III., Problem V., and to the symbols on the same and following page, it will be readily perceived, that by drawing a perpendicular from a on N B, (which is not shewn in the diagram) that $a\,q = a - 2\,x_a$, whence the area in acres of the trapezoid N $a\,q\,p = \frac{1}{2}$ (N p — $a\,q$) $p\,q = \dfrac{(a - \alpha\,x)\,x}{l}$ in a similar way the area of B $b\,s\,r$ is found $= \dfrac{(c - \beta\,x)\,x}{l}$, the area of the rect-

angle $p\,q\,s\,r$ being obviously $= \frac{b\,x}{l}$. By multiplying these three areas by their respective values, and putting their sum equal to the required value V′, their results

$$\frac{(a - \alpha\,x)\,m\,x + (c - \beta\,x)\,o\,x + b\,n\,x}{l} = \mathrm{V}'; \text{ whence}$$

$(\alpha\,m + \beta\,o)\,x^2 - (a\,m + b\,n + c\,o)\,x + l\,\mathrm{V}' = 0$; and, by solving this quadratic for the reciprocal of x, there results

$$x = \frac{\cdot 2\,l\,\mathrm{V}'}{a\,m + b\,n + c\,o + \sqrt{(a\,m + b\,n + c\,o)^2 - 4\,(a\,m + \beta o)\,l\,\mathrm{V}'}} = p\,q \text{ or } r\,s.$$

NOTE. There is an error in the formula at page 120, which is noticed in the List of Errata.—The method of adopting this formula to a greater or less number of qualities of land is sufficiently clear, from the different modifications of the preceding formula in Cor. I. and II.

PROBLEMS, ORIGINAL AND SELECT,
FOR THE EXERCISE OF STUDENTS.

I. The side of an equilateral triangle is $= a$, and its area $=$ A; prove that

$$\mathrm{A} = \tfrac{1}{4}a^2\sqrt{3}.$$

II. The base of an isosceles triangle is $= a$, one of its equal sides $= b$, and its area A; prove that

$$\mathrm{A} = \tfrac{1}{4}a\sqrt{(2\,b + a)\,(2\,b - a)}.$$

III. In a triangle are given the perpendicular $= p$, the angles opposite the perpendicular $= \alpha$, and β, and consequently the third angle $= \gamma$; prove that

$$\mathrm{A} = \frac{p^2 \text{ sine } \gamma}{2 \text{ sine } \alpha \text{ sine } \beta}.$$

IV. A B C D is a trapezium, in which the angles at A and C are equal, and A B $= a$, B C $= b$, C D $= c$, D A $= d$, the half sum of all the sides $= s$, and the area $=$ A; then prove that

$$\mathrm{A} = \frac{a\,d + b\,c}{a\,d - b\,c}\sqrt{s\,(s - a - c)\,(s - b - c)\,(s - c - d)}.$$

V. In a trapezium A B C D, are given two opposite sides A B $= a$, C D $= b$, the angles at A, B, and C respectively $= \alpha$, β, and γ, and consequently the fourth angle D $= \delta$; then prove that

$$\mathrm{A} = \frac{a^2 \text{ sine } \alpha \text{ sine } \beta - b^2 \text{ sine } \gamma \text{ sine } \delta}{2 \text{ sine } (\alpha + \beta)}.$$

VI. In a trapezium A B C D, are given the three sides

C D = a, B A = b, A D = c, and the angles at A and B respectively = α and β; then prove that

$$A = \tfrac{1}{2}\left\{a\,b \sin\beta + b\,c \sin\alpha - a\,c \sin(\alpha+\beta)\right\}.$$

VII. In a pleasure ground A B C D, known to be rectangular, only the following dimensions could be taken, on account of obstructions from buildings, shrubberies, and ponds, viz, the distance A E to the point where the perpendicular B E falls on the diagonal A C, and the prolongation E F of B E till it meet the side C D; it is required to find the area of the pleasure ground, when B E = 32, and E F = 4 chains.

Ans. 64 acres.

VIII. A large building is known to be of a square form, but no one of its sides could be measured, on account of obstructions from other buildings; however, from a point P three streets diverged irectly to its three nearest angles A, B, and C. Now P A = 60, P B = 40 and P C = 70 yards; required the side of the building.

IX. Within a rectangular garden, the length of which is 4 and its breadth 3 chains, there is a piece of water in the form of a trapezium, the opposite angles of which are in a direct line with those of the garden, the distances of these opposite angles, taken in succession, are 20, 25, 40 and 45 yards; required the area of the water. *Ans.* 960 square yards.

X. Given the three perpendiculars of a triangle, from the angular points to their opposite sides, 10, 11, and 12 chains, to find the area of the triangle. *Ans.* 7*a*, 0*r*. 4½*p*.

XI. There are given the four sides of a trapezium, and a line joining given points in two of its opposite sides, from which it is required to construct the figure.

XII. The five sides a, b, c, d, and e of an irregular field are given, in which three of the angles are equal but not given; from these *data* it is required to lay down the field.

NOTE. The solutions of the two last Problems are obtainable by cubic equations. The former was proposed by the author, in the *Gentleman's Diary* for 1838, it having occurred in his practice of town surveying for railway purposes. The latter was proposed by *B. Gompertz, Esq., F.R.S., &c.*, in the *Gentleman's Mathematical Companion;* to which he gave a solution in a concise, novel, and ingenious manner by his *Principles of Imaginary Quantities:* other solutions by the ordinary methods were also given to the same Problem.

(DIVISION OF LAND.)

XIII. In a triangle ABC, AB = a, BC = b, CA = c, and

from D, a given point in A B, the distance D B = d, D E a line meeting B C and dividing the triangle, so that $\triangle$ A B C : $\triangle$ B D E :: m : n; then prove that

$$D E = \frac{a b n}{d m}.$$

But if it be found from the calculation that B E is greater than B C, and that the divisional line will meet A C in some point F; then prove that

$$A F = \frac{a c (m - n)}{m (a - d)}.$$

XIV. A B C D is a given trapezium, and T a given point in A D, from which a line T E is drawn to meet B C in E, so that trapezium A B C D : trapezium D C E T :: m : n. Now, since the trapezium A B C D is given, if the sides A D, B C be prolonged till they meet in Z, the point Z will also be given; therefore, put A Z = a, B Z = b, D Z = c, C Z = d, and T Z = f; then prove that

$$E Z = \frac{n (a b - c d)}{m f} + \frac{c d}{f}.$$

XV. It is required to divide a given trapezium into four equal parts, by two right lines perpendicular to one another.

NOTE. This Problem was proposed, the author believes, in one of the *Diaries*, many years ago, where its solution may be seen.

(RAILWAY ENGINEERING.)

XVI. Let δ be distance between the tangent points of a serpentine curve, consisting of two circular arcs of equal radii r, α and β the angles that δ makes with the tangents, and σ = arc to $\frac{1}{2}$ (cos. α + cos. β); then prove that

$$r = \frac{\delta}{\text{sine } \alpha + \text{sine } \beta + 2 \text{ sine } \sigma}.$$

XVII. Find the distances of the tangent points to the point of contrary flexure of the curve, in the last Problem, and the angles that these distances make with the tangents to the curve, and give the requisite formula.

XVIII. The widths of a laterally sloping railway cutting from the centre of the line are expressed generally by the following formula,

$$w = \frac{b s}{l \pm r h},$$

the positive sign being used for the width measure down the slope, and the negative one for that up the slope, in which

formula $b = \frac{1}{2}$ width of the cutting, assuming its surface to be level, $h =$ differences of level readings at the distances s and l on the slope and level, and r the ratio of the slopes. See figure to Prob. II. Chap. III.

XIX. Let a and b be the depth of a railway cutting to the intersection of the slopes, $l =$ length of the cutting, $\omega = \frac{1}{2}$ bottom width, all in feet, and $r =$ ratio of the slopes, the surface of the cutting being assumed to be horizontal; then prove that the content of the cutting in cubic yards is

$$\frac{l\,r}{81}\left(a^2 + a\,l + b^2 - \frac{3\,\omega^2}{r^2}\right).$$

XX. When a and b are the depths of a horizontal cutting to the formation level, and the other dimensions the same as in the last Problem; then prove that the content of the cutting in cubic yards is

$$\frac{l\,r}{81}\left\{a^2 + a\,b + b^2 - \frac{3\,\omega}{r}(a+b)\right\}.$$

XXI. Let A and B be the areas of the cross sections of a cutting to the intersection of the slopes, d the area of the end of the prism below the formation level, (see fig. p. 189) and l the length of the cutting; then prove that the content in cubic yards is

$$\frac{l}{81}(\mathrm{A} + \mathrm{B} + \sqrt{\mathrm{A}\,\mathrm{B}} - 3\,d).$$

XXII. Let the dimensions be the same as in Prob. XIX.; then prove that the error in defect of the method of finding the content of a cutting by using the mean depths is

$$\frac{l\,r}{324}(a-b)^2.$$

XXIII. Let the dimensions be the same as in Prob. XXI.; then prove that the error in excess of the method of finding the content of a cutting by using mean areas is

$$\frac{l}{162}(\sqrt{\mathrm{A}} - \sqrt{\mathrm{B}})^2.$$

XXIV. Prove the formula at p. 197, for finding the error per cent. of Mr. Bashforth's method of finding the contents of railway cuttings from sectional areas.

NOTE. The demonstrations of the nine last Problems are given in *Baker's Railway Engineering*.

CENTRIFUGAL FORCE OF TRAINS IN RAILWAY CURVES.

Since all moving bodies have a tendency to continue their motion in a direct line, from this cause the carriages of a rail-

way train of great velocity are strongly urged towards the outer rail, and would ultimately be driven off the rails, were it not for the flanges of the wheels and the conical inclination of their tire.

Let F = centrifugal force thus generated, W = weight of the train, V = its velocity, R = radius of the curve, in which the train moves, and g the force of gravity at the earth's surface; then by Dynamics

$$F = \frac{W\ V^2}{g\ R}.$$

EXAMPLE.

1. When R = ¼ a mile = 2640 feet, V = velocity = 30 miles per hour = 44 feet per second, and g = 32⅛ feet = velocity of a body falling from rest, at the end of a second; then

$$F = \frac{W \times 44^2}{32\frac{1}{8} \times 2640} = \frac{22}{9651}\ W = \text{nearly } \frac{1}{44}\ W,$$

that is, the force that urges the train to quit the curve is $\frac{1}{44}$ of its whole weight, in this case.

2. When V = 60 miles per hour = 88 feet per second, and R the same as in Example 1; then

$$F = \frac{W \times 88^2}{32\frac{1}{8} \times 2640} = \text{nearly } \frac{1}{11}\ W,*$$

that is the force, in this case, is $\frac{1}{11}$ of the weight of the train. Hence it may be perceived how extremely dangerous high velocities are in curves of small radius.

* This great amount of centrifugal force, in curves of small radius, would be very much increased by the high velocities, which some are sanguine enough to expect as likely to be attained on railways; since this force varies as

$$\frac{V^2}{R} \text{ or as } V^2$$

for the same curve: thus for a velocity of 120 miles per hour, on a curve of ¼ a mile radius, we shall have

$$f = \frac{W \times 176^2}{32\frac{1}{8} \times 2640} = \frac{4}{11}\ W,$$

that is, the centrifugal force is, in this case, more than ⅓ of the whole weight of the train; while for curves of 1 mile radius, which are very common in railways, $f = \frac{2}{11}$ W, or nearly ⅕ of the weight of the train. It must, therefore be evident that a velocity of 120 miles per hour, or even one of 90 miles per hour, must be extremely dangerous, especially on an embanked curve, should any accident throw the train off the line, which is often the case with the present velocities. Moreover, the resistance of the air, which varies as V^2, must be considerably augmented by high winds opposed to the direction of a train of these great velocities; while its engine would require a power greatly superior to those now in use.

3. When the radius is = 1 mile = 5280 feet, and V the same as in Example 2 ; then

$$F = \tfrac{1}{2\frac{1}{2}} W.$$

This force, except in curves of very small radius, is counteracted by the conical inclination of the tire of the wheels of the engine and its train. This inclination with the lateral play of the flanges of the 2 wheels of about $\frac{1}{4}$ an inch on each side, and the centrifugal force urging the train towards the outer rail, when moving in a curve, increase the diameter of the outer wheel and diminish that of the inner one, which causes the train to roll on a conical surface, thus necessarily producing a centripetal force to counteract the tendency of the train to leave the curve. However, in curves of very small radius, the centripetal force thus generated, does not sufficiently counteract the centrifugal force, a proper superelevation of the exterior or outer rail being required for this purpose ; for determining which *Pambour* has given in his work on *Locomotive Engines*, the folloming

FORMULÆ FOR THE SUPER-ELEVATION OF THE EXTERIOR RAIL.

Let V = velocity of the train, R = radius of railway curve, R′ = radius of the curve that the train would describe in consequence of conical shape of the tire of the wheels, and the centrifugal force impelling the train outward, and enlarging the diameter of the outer, and diminish that of the inner wheel, e = gauge of rails, g = force of gravity, and x = superelevation of outer or exterior rail ; then

$$x = \frac{e\,V^2}{9}\left(\frac{1}{R} - \frac{1}{R'}\right),$$

in which

$$R' = \frac{d\,e\,n}{4\,\Delta},$$

d being = outer diameter of the wheels, Δ = deviation of the wheels, and $\frac{1}{n}$ = the inclination of the tire.

Concise and, he trusts, clear demonstrations of the above formulæ are given by the author, in his *Railway Engineering*.

By solving these formulæ for some of the usual cases, *Pambour* produces the following

TABLE OF THE SUPER-ELEVATION TO BE GIVEN TO THE EXTERIOR RAIL IN CURVES.

Designation of the Waggons and the Way.	Radius of the Curve in Feet.	Super-elevation to be given to the Rail in Inches, the Velocity of the motion in Miles per hour being:—		
		10 Miles.	20 Miles.	30 Miles.
Waggon with wheels 3 feet in diameter. Gauge of way, 4·7 feet. Play of the waggons on the way, 1 inch. Inclination of the tire of the wheels, 1 in 7.	250	1·14	5·60	12·99
	500	0·57	2·83	6·56
	1000	0·29	1·43	3·30
	2000	0·15	0·71	1·65
	3000	0·10	0·47	1·10
	4000	0·07	0·36	0·83
	5000	0 06	0·28	0·66

The correctness of the above results is pretty generally conceded. It must, however, be considered, that it is extremely difficult, if not impossible, to realize in practice, the precise conditions and proportions determined by these important formulæ; as accidental depressions and enlargements of gauge of part of the rails, as well as many other matters that cannot be subjected to calculation, will unavoidable derange these results.

The reader, who wishes for further information on these subjects, may consult *Hann's Tredgold on the Steam Engine; Moseley's Mathematical Principles of Architecture and Engineering; Hann's Treatise on the Steam Engine, with Practical Rule for the use of Engineers; and Tate's Exercises in Mechanics and Theory of the Steam Engine, in addition to Barlow's and Pambour's Works, already noticed.*

APPENDIX.

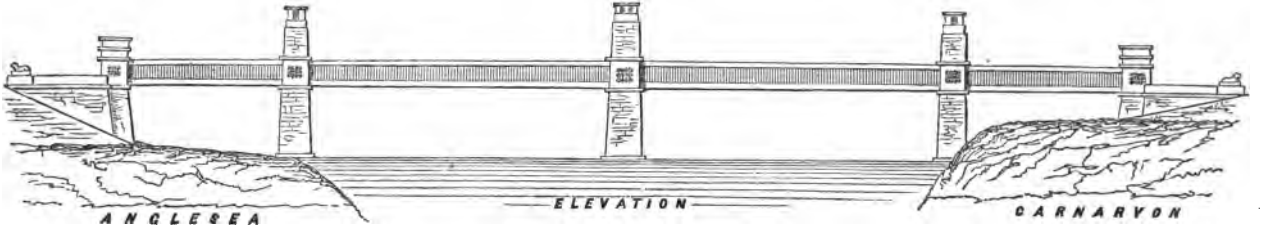
RAILWAY VIADUCTS OR BRIDGES.

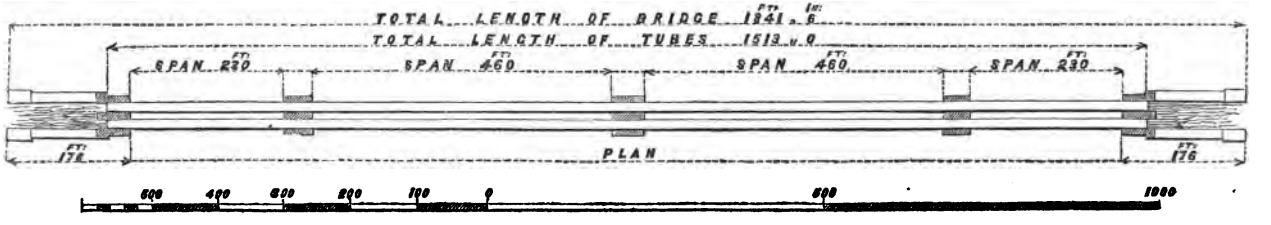
The extensive lines of railway lately constructed, and now in progress, present numerous viaducts, some of which are stupendous as well as magnificient works, and some of a truly novel character. The dimensions of the chief specimens are here given.

TUBULAR AND OTHER IRON GIRDER BRIDGES.

The Britannia tubular bridge. This structure, combining unparallelled magnitude, strength, and novelty, forms one of the viaducts of the *Chester and Holyhead Railway.* It crosses the Menai Straits, uniting the shores of the mainland of Wales and the Isle of Anglesea. It consists of two rectangular tubes, each 1513 feet in length, or $\frac{2}{7}$ of a mile, 26 feet in average depth, and 14 feet 8 inches in width, the internal depth and width being respectively reduced by the construction to 22 and 14 feet. Each tube has four spans, and consequently three piers or towers, exclusive of the abutments. The two middle spans, in each tube, are each 460 feet, and the two end spans each 230 feet, exclusive of the widths of the towers, which support the tubes at a height of 102 feet above high water mark, the whole height of the middle tower being 200 feet above high water mark, or 230 feet from its foundation. The parts of the tubes forming the middle spans were 472 feet in length, previous to their being united, and weighed upwards of 1600 tons, and were raised to their present lofty position by hydraulic presses worked by steam engines, thus leaving the navigation of the Menai Straits uninterrupted.

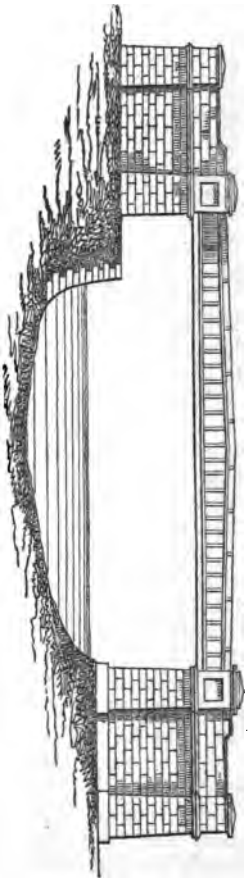
"It is seldom," say Mr. G. D. Dempsey, "that the invention of works of new design and skilful mechanical arrangement is due entirely to one mind, any more than their construction is due to one pair of hands: hence great difficulty arises in assigning to each contributor his fair share of merit in their production. It must, however, be admitted, that to Mr. R. Stephenson alone we are in this instance indebted for the original suggestion; and with this admission, we have endeavoured to avoid any attempt to judge of the precise claims of


ANGLESEA
ELEVATION
CARNARVON


TOTAL LENGTH OF BRIDGE 1841 ft. 6 in.
TOTAL LENGTH OF TUBES 1513 ft. 0
SPAN 230 ft.
SPAN 460 ft.
SPAN 460 ft.
SPAN 230 ft.
176 ft.
PLAN
176 ft.
600
400
200
100
0
600
1000

the two eminent men, whose joint labours have produced the Britannia and Conway Tubular Bridges. That these great works owe their design and construction to their *joint* labours is clearly evident, and, we respectfully submit, amply sufficient to justify the record of the two names of Robert Stephenson and William Fairbairn in an honourable and enduring association."

The Conway tubular bridge, in the same line of railway, preceded the Britannia, having been raised in 1848. It consists of one span only of 400 feet, clear width: the height of the tubes above the level of high water is inconsiderable, when compared with the Britannian tubes, being only 18 feet. The bridge thus consists of two tubes only, each weighing 1300 tons. It is erected close beneath the ancient wall of Conway Castle, its abutments being of strong masonry, the design of which are in harmony with that of the castle.



IRON TUBULAR GIRDER BRIDGES.

The first tubular girder bridge designed and constructed by Mr. Fairbairn was for the purpose of carrying the Blackburn and Bolton Railway over the Leeds and Liverpool Canal. The span of this bridge is 60 feet, the two lines of rails being carried between three parallel girders.—The cellular work in girders of this kind constitutes their great strength, combined with comparative lightness, the same kind of cellular work being introduced both in the top and bottom of the tubes of the Britannia and Conway Bridges.

The Gainsborough tubular girder bridge is the largest yet constructed of this kind. It forms the viaduct of the Manchester, Sheffield and Lincolnshire Railway over the Trent; and consists of two spans, each 154 feet wide, with a central pier and abutments of masonry, and two end arches each 40 feet span. This bridge crosses the river obliquely,

the abutment and pier are therefore placed at an angle of 50° with the longitudinal direction of the girders, which are two in number, and of uniform depth throughout, between which the double line of railway is carried. This elegant specimen of wrought ironwork was designed and executed by Mr. W. Fairbairn, whose name is so honourably connected with the Britannia and Conway Tubular Bridges.

BRICK AND STONE VIADUCTS.

The brick viaduct at Maidenhead, constructed by Mr. Brunel over the Thames on the Great Western Railway, is one of the finest specimens of that material. It consists of two elliptical arches, each 128 feet span, with a rise of 24¼ feet; the pier between the two arches is 30 feet in width; each arch, at the crown, is 5¼ feet in thickness, which increase gradually towards the abutments. Besides these two splendid arches, there are eight others, each of 28 feet span, four on each bank of the river.

The stone viaduct over the Ouse, near York, on the Great North of England Railway, is another specimen combining great strength and elegance of construction. It consists of three arches, each 66 feet span, the width of the arches is 28½ feet, their thickness at the keystone 3½ feet, which gradually increases towards the springing, and the piers are 10 feet in thickness.

The Vale Royal stone viaduct, over the river Weaver, on the Grand Junction Railway, has five arches, each 68 feet span, and 60 in height; the whole length of the viaduct is 456 feet. In the same railway, over the Mersey and Travell Canal, there is a viaduct of 12 stone arches, the central two are each 75 feet span, and the rest from 40 to 14 feet.

The Quaker's Yard viaduct, over the Taff, in the *Taff Vale Railway*, has six arches; its height above the river is 100 feet, and its length 600 feet. There is another viaduct in the same line, at the conflux of the Rhondda and Taff, having an arch 100 feet in span, and 60 in height.

The Ouse Valley viaduct, in the London and Brighton Railway, is 1440 feet in length; it has 37 brick arches of 30 feet span, its height varying from 40 to 96 feet.

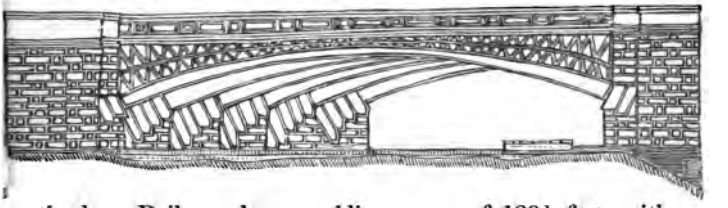
The London and Greenwich Railway, and *the London and South Western Extension* are continuous viaducts; the former consists of more than 1000 brick arches, each 18 feet span, and 24 in height; the arches in the latter are a little wider, having, besides, several elegant oblique iron arches, for

the purpose of crossing Westminster Road and other principal streets. The former is about four miles in length, and the latter nearly two miles.

OBLIQUE OR SKEW BRIDGES.

The Brampton viaduct, over the Gelt, in the Newcastle and Carlisle Railway, crosses the Gelt and a public road, at the height of 80 feet above the bed of the river. It consist of three stone arches of 33 feet in direct span, which are built at an inclination of 45°, thus making their oblique span nearly 50 feet each.

The Fairfield Street bridge, in the Manchester and Birmingham Railway, has an oblique span of 129½ feet, with a rise of 12 feet, and a direct width of 31 feet. It is composed of six ribs of iron, projecting before each other 13 feet, on independent abutments. Open wrought stone parapets crown this viaduct, presenting an agreeable and novel appearance. This bridge was designed by Mr. G. W. Buck, being remarkable for its acute angle, which is only 24½°. The weight of the iron in the six ribs or girders is 540 tons: the remainder of the viaduct consist of brick arches 45 feet in span.



NOTE. Other lines of railway present a great variety of these bridges, both of brick and iron; some of which display great elegance of construction. This kind of bridges, it is said, was first introduced by Mr. Chapman, as canal aquaducts in Ireland.

HIGH LEVEL BRIDGE, NEWCASTLE UPON TYNE.

This bridge is of cast iron, and forms the viaduct of the Great North of England Railway across the deep valley of the Tyne. In addition to the elegance and magnitude of this work, it has a common carriage way suspended beneath the railway, thus giving at a decided claim to novelty of construction.

The number of its iron arches are six. Each arch consists of four cast iron ribs, having a span of 125 feet, rising 17½ in the centre. The ribs are disposed in pairs: one pair on each side of the carriage road. The carriage road in 20⅓ feet wide. The foot roads (one between each pair of ribs) 6½ feet wide.

Total width of railway 33 feet. Weight of iron in each arch 576 tons; in the whole structure about 5000 tons. Height from high water to level of rails 108½ feet; to suspended carriage road 85 feet; total height from bottom of river 128 feet. Length of bridge from the Castle Garth to Gateshead, inclusive of cast iron approaches, 1337½ feet.

WOODEN BRIDGES.

The Ouseburn viaduct, in the Newcastle and North Shields Railway, has nine arches, of which two at the ends are stone, the other seven are of wood, on stone piers; the three central arches are each 116 feet span, and the two others each 100 feet. The whole length of the viaduct is 756 feet, and its height above the stream in the centre is 180 feet.

The Willington Dean viaduct, in the last mentioned line, consists of seven wooden arches, of which five have a span of 120 feet each, and the two exterior ones 115 feet each. The whole length of the viaduct is 1050 feet, and its height 82 feet.

Wooden viaducts, on a smaller scale, abound in various lines of railway in this kingdom.—Wooden bridges on the *lattice principle* (first suggested by Mr. G. Smart, about 25 years ago, as applicable both to iron and wooden bridges), abound in the United States of America. One of them, over the Susquehannah at Columbia, has 29 spans, each 200 feet wide, the whole viaduct being about 1¼ miles in length. These viaducts can not be anticipated to be of long duration. The *lattice principle* has been considerably improved, and employed in wrought iron bridges by Mr. R. B. Osborne. This principle has been still further extended by Mr. W. C. Harrison, by adding tubular frames to the lattice work above and below, the upper one being bent like an arch. In a design made by him for a viaduct of this kind to carry a railway over the river Ouse, the span is 170 feet, and the rise of the arched tube above the horizontal one is 15 feet. The arched tube to be of wrought iron plates ½ an inch in thickness, and its section to be 4 feet in dept and 3 in width, the horizontal tube to be a little less. For a double line of railway three of these bow frames are required.

No. 1.

Offsets at the end of the first chain from tangent point of railway curves.

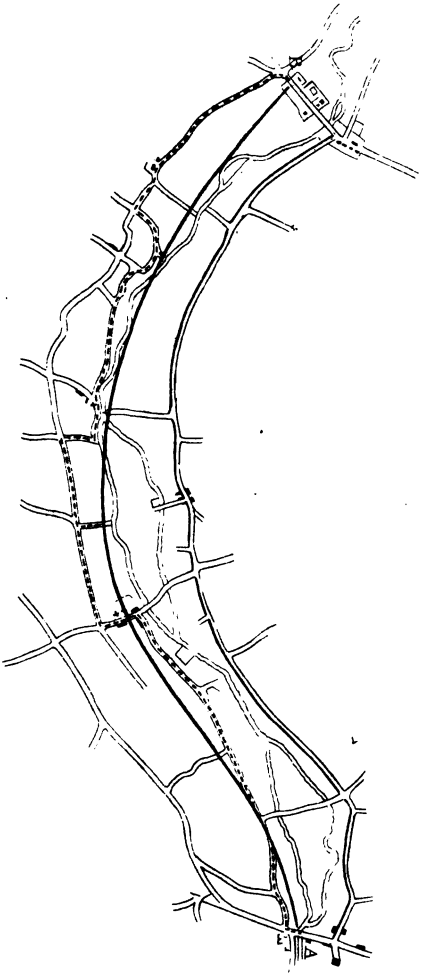
Radius of curve in chns.	Offsets in inches and decimals.	Radius of curve in chns.	Offsets in inches and decimals.	Radius of curve in chns.	Offsets in inches and decimals.	Radius of curve in chns	Offsets in inches and decimals.
5	79·2000	54	7·3333	105	3·7714	186	2·1290
6	66·0000	55	7·2000	106	3·7358	188	2·1064
7	56·8714	56	7·0714	108	3·6667	190	2·0842
8	49·5000	57	6·9473	110	3·6000	192	2·0625
9	44·0000	58	6·8276	112	3·5352	194	2·0412
10	39·6000	59	6·7118	114	3·4736	195	2·0307
11	36·0000	60	6·6000	115	3·4435	196	2·0204
12	33·0000	61	6·4918	116	3·4138	198	2·0000
13	30·4615	62	6·3871	118	3·3559	200	1·9800
14	28·2857	63	6·2857	120	3·3000	205	1·9317
15	26·4000	64	6·1875	122	3·2459	210	1·8857
16	24·7500	65	6·0923	124	3·1935	215	1·8418
17	23·2941	66	6·0000	125	3·1680	220	1·8000
18	22·0000	67	5·9104	126	3·1428	225	1·7600
19	20·8421	68	5·8235	128	3·0937	230	1·7217
20	19·8000	69	5·7391	130	3·0461	235	1·6851
21	18·8571	70	5·6571	132	3·0000	240	1·6500
22	18·0000	71	5·5774	134	2·9552	245	1·6163
23	17·2174	72	5·5000	135	2·9333	250	1·5840
24	16·5000	73	5·4246	136	2·9117	255	1·5529
25	15·8400	74	5·3513	138	2·8645	260	1·5231
26	15·2307	75	5·2800	140	2·8285	265	1·4943
27	14·6666	76	5·2105	142	2·7887	270	1·4667
28	14·1428	77	5·1428	144	2·7500	275	1·4400
29	13·6551	78	5·0769	145	2·7310	280	1·4143
30	13·2000	79	5·0126	146	2·7123	285	1·3995
31	12·7742	80	4·9500	148	2·6756	290	1·3655
32	12·3750	81	4·8889	150	2·6400	295	1·3423
33	12·0000	82	4·8292	152	2·6052	300	1·3200
34	11·6470	83	4·7711	154	2·5714	305	1·2983
35	11·3142	84	4·7143	155	2·5548	310	1·2774
36	11·0000	85	4·6588	156	2·5384	315	1·2571
37	10·7026	86	4·6046	158	2·5063	320	1·2375
38	10·4210	87	4·5517	160	2·4750	325	1·2184
39	10·1538	88	4·5000	162	2·4444	330	1·2000
40	9·9000	89	4·4496	164	2·4146	335	1·1821
41	9·6588	90	4·4000	165	2·4000	340	1·1647
42	9·4285	91	4·3516	166	2·3855	345	1·1478
43	9·2093	92	4·3043	168	2·3571	350	1·1314
44	9·0000	93	4·2581	170	2·3294	355	1·1155
45	8·8000	94	4·2128	172	2·3023	360	1·1000
46	8·6087	95	4·1684	174	2·2758	365	1·0809
47	8·4255	96	4·1250	175	2·2628	370	1·0703
48	8·2500	97	4·0825	176	2·2500	375	1·0560
49	8·0816	98	4·0408	178	2·2248	380	1·0421
50	7·9200	99	4·0000	180	2·2000	385	1·0285
51	7·7647	100	3·9600	182	2·1758	390	1·0154
52	7·6154	102	3·8824	184	2·1521	395	1·0025
53	7·4717	104	3·8077	185	2·1405	400	·9900

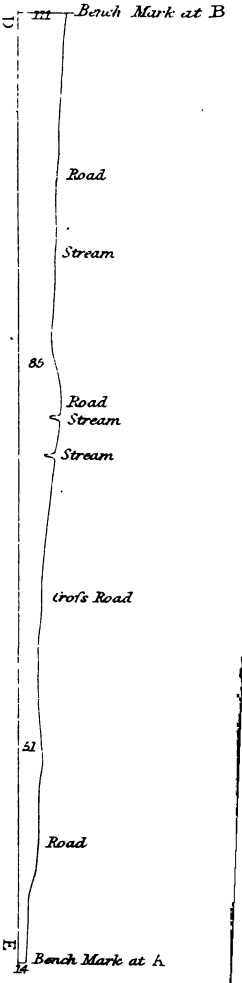
No. 2.—Difference between apparent and true level for distances in chains. Correction in decimals of feet.

Dist. in chains.	For curvature.	For refraction.	For curvature and refraction
3½	·001	·000	·001
4	·002	·000	·002
4½	·002	·000	·002
5	·003	·000	·003
5½	·003	·000	·003
6	·004	·000	·004
6½	·004	·001	·003
7	·005	·001	·004
7½	·006	·001	·005
8	·007	·001	·006
8½	·007	·001	·006
9	·008	·001	·007
9½	·009	·001	·008
10	·010	·001	·009
10½	·011	·002	·009
11	·013	·002	·011
11½	·014	·002	·012
12	·015	·002	·013
12½	·016	·002	·014
13	·018	·002	·016
13½	·019	·003	·016
14	·020	·003	·017
14½	·022	·003	·019
15	·023	·003	·020
15½	·025	·004	·021
16	·027	·004	·023
16½	·028	·004	·024
17	·030	·004	·026
17½	·032	·005	·027
18	·034	·005	·029
18½	·036	·005	·031
19	.038	·005	·033
19½	·040	·006	·034
20	·042	·006	·036
20½	·044	·006	·038
21	·046	·007	·039
21½	·048	·007	·041
22	·050	·007	·043
22½	·053	·007	·046
23	·055	·008	·047
23½	·057	·008	·049
24	·060	·009	·051
24½	·062	·009	·053
25	·065	·009	·056
25½	·068	·010	·058
26	·070	·010	·060
26½	·073	·010	·063
27	·076	·011	·065
27½	·079	·011	·068
28	·082	·012	·070
28½	·085	·012	·073
29	·088	·012	·076
29½	·091	·013	·078
30	·094	·013	·081

No. 3.—Difference between apparent and true level for distances in miles. Correction in feet and decimals.

Dist. in miles.	For curvature.	For refraction.	For curvature and refraction.
¼	·04	·01	·03
½	·17	·02	·15
¾	·37	·05	·32
1	·67	·09	·58
1¼	1·50	·21	1·29
2	2·67	·38	2·29
2½	4·17	·60	3·57
3	6·00	·86	5·14
3½	8·17	1·17	7·00
4	10·67	1·52	9·15
4½	13·55	1·93	11·62
5	16·67	2·38	14·29
5½	20·18	2·88	17·30
6	24·01	3·43	20·58
6½	28·18	4·03	24·15
7	32·68	4·67	28·01
7½	37·52	5·36	32·16
8	42·69	6·10	36·59
8½	48·19	6·88	41·31
9	54·02	7·72	46·30
9½	60·20	8·60	51·60
10	66·70	9·53	57·17
10½	71·52	10·22	61·30
11	80·69	11·53	69·16
11½	88·18	12·59	75·59
12	96·00	13·71	82·29
12½	104·17	14·88	89·29
13	112·67	16·09	96·58
13½	121·50	17·36	104·14
14	130·67	18·67	112·00
14½	140·17	20·02	120·15
15	150·00	21·43	128·57
15½	160·17	22·88	137·29
16	170·67	24·38	146·29
16½	181·50	25·93	155·57
17	192·67	27·52	165·15
17½	204·17	29·17	175·00
18	216·00	30·86	185·14
18½	228·17	32·58	195·59
19	240·67	34·38	206·29
19½	253·50	36·21	217·29
20	266·67	38·07	228·60
21	294·00	42·00	252·00
22	322·67	46·09	276·58
23	352·67	50·38	302·29
24	384·00	54·86	329·14
25	416·67	58·09	358·58
26	450·67	64·38	386·29
27	486·00	69·43	416·57
28	522·67	74·67	448·00
29	560·67	80·09	480·58
30	600·00	85·71	514·29


B
A


Bench Mark at B
Road
Stream
86
Road
Stream
Stream
Cross Road
51
Road
Bench Mark at A
14

G